Essays on Child Care Subsidies

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Dedication

To my parents, Bronwyn Frame and GianCarlo Moschini.
Mamma e Papà, grazie per i vostri consigli. Mi hanno aiutato a continuare il mio cammino in questa selva oscura.

Abstract

This dissertation consists of three chapters. In the first chapter, I review the literature on child care subsidies, discussing three main strands to which I contribute: general equilibrium models of skill investment, estimations of skill accumulation technologies, and evaluations of child care subsidy programs. In the second chapter, I lay out a model of child care subsidies in an environment where one- and two-parent families form endogenously and then altruistically invest in their children's skill, using both their own time and time purchased on the market in the form of child care. I compare welfare gains under three possible designs for a child care subsidy, which differ in the pool of those entitled to receive the subsidy (the eligible). In the third chapter of my dissertation, I go into detail on the estimation of the skill accumulation technologies used by one- and two-parent families to invest in their children. In this section I describe my main data source (the Early Childhood Longitudinal Study, Birth Cohort), the econometric methods used to find point estimates, the implications of my findings for my results, and how my findings compare with those outstanding in the literature.

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Chapter 1

Literature Review

The model presented and estimated in chapters 2 and 3 of this dissertation is related to three main strands of the literature on human capital. First, studies of skill investment subsidies in general equilibrium; second, estimations of skill accumulation technologies; and third, evaluations of child care subsidy programs. In this chapter, I provide context for the model I develop within these three strands.

I first provide a broad overview on the outstanding studies in macroeconomics that addresses skill accumulation subsidies in general equilibrium, with a focus on those that consider parental investment in children's skill. Here I want to highlight the way in which the model in Chapter 2 of this dissertation departs from the standard framework in the literature, and why this departure is a useful one. Second, because the most important primitive for my analysis are the two skill accumulation technologies, I give a short discussion of the literature on the estimation of these functions. In the course of this discussion I make the case for my choice of data set relative to the other similar estimations in the literature. Finally, I briefly review the large body of work estimating the treatment effects of child care subsidies on child outcomes, using data on both randomized controlled trials and natural experiments. This discussion is meant to highlight both the fact that the sign and magnitude of these effects is not a settled question, and that a structural model can be useful precisely because it makes explicit general equilibrium feedbacks from such interventions.

1.1 Skill Investment Subsidies in General Equilibrium

The focus of the macroeconomic literature on intergenerational transfers is usually to decompose intergenerational elasticity (IGE) in earnings or to examine the sources of income inequality. Studies then examine welfare gains from subsidies directed at different points in the lifecycle (e.g., Restuccia and Urrutia (2004), Lochner and Monge-Naranjo (2011), Guner, Kaygusuz, and Ventura (2016), Gayle, Limor, and Soytas (2017), Daruich (2017), Lee and Seshadri (2018)). The reason the IGE is a statistic of interest is that environments with a higher IGE are more risky ex ante. A utilitarian social planner will see an increase in welfare from subsidies that lower the IGE. A similar logic motivates the focus on income inequality.

To generalize away from specific statistics and toward a common question they are used to address, general equilibrium studies of skill investment subsidies that incorporate family investments are interested in the degree to which heterogeneity in outcomes at adulthood is due to individual-level characteristics at birth as opposed to characteristics of the family into which a child is born.

The standard framework used in these studies is one with overlapping generations, where families invest in the next generation of their dynasty with time and money. The motive for these investments can be altruistic (parents internalize the actual outcomes of their children) or paternalistic (parents only care about the transfers, not their effect). In such studies, families usually consist of one-parent and one-child, while targeted moments are estimated on samples of two-parent families.

My model extends a common framework found in these studies: among macroeconomic studies of the welfare effects from policies that distort parental investment in child skill, a distinctive feature of my paper is that it incorporates a marriage market. This allows one- and two-parent families to arise endogenously in equilibrium. In my model, the endogenous sorting of adult types across family structures is distinct from the technologies those structures use to invest in their children. In this way, I separate the endogenous composition effect from the exogenous primitive differences across family structures in terms of the skill accumulation technologies parents use to invest in their children. This allows early childhood education policy and the skill distribution within and across family structures to interact in equilibrium.

This is an important interaction to endogenize for the following reason. The skill accumulation technology that parents use to invest in their children implies, for a given set of parameter values, an elasticity of the price of investment with respect to the price of child care. This responsiveness to the price of child care is allowed to be heterogenous in my model because I incorporate two distinct skill accumulation technologies. Without endogenous family formation, the family structure with higher sensitivity to the subsidy would be forced to persist in equilibrium, perhaps inflating gains from the program. Allowing the most sensitive households to no longer exist once the subsidy has been implemented gives a more realistic sense of the potential gains from such interventions.

The family formation channel in my model is similar to that of Abbott, Gallipolli, Meghir, and Violante (2018), which examines how subsidies directed at college students (rather than parents with young children) can effect the composition of marriages. However, in that study single-parent families do not exist in equilibrium.

1.2 Estimations of Skill Accumulation Technologies

The skill accumulation technology is a function which governs the ability of investments to affect the stock of skill, and the relative contribution of different kinds of investments (such as money and time) to skill accumulation. There are many ways to formulate this technology (see Todd and Wolpin (2003) for an overview of this modelling decision). Studies that use skill accumulation technologies commonly specify functions are constant elasticity of substitution (CES) or Cobb-Douglas; typical arguments include the current stock of skills, parental skills, local environments, and investment in terms of money and time.

There are two surveys commonly used to estimate these technologies: the Children of the National Longitudal Survey of Youth 1979 (Children of the NSLY79) and the Panel Study of Income Dynamics, Child Development Supplement (PSID CDS). The design of these surveys limits the kinds of functional forms that are estimable using the information they provide. For example, in the Children of the NLSY79 there is no time diary and little information on fathers.

To my knowledge, only Del Boca, Flinn, and Wiswall (2014) and Abbott (2018), both using the PSID CDS, incorporate father's time into the skill accumulation technology as a separate input. To do so, these studies capitalize on child time diaries which report on activities with both members of a parenting couple. These two studies differ in the functional form they estimate: Del Boca, Flinn, and Wiswall (2014) uses a Cobb-Douglas functional form, while Abbott (2018) estimates rather than assumes the complementarity parameter by estimating a constant elasticity of substitution (CES) functional form. Both focus on two-parent families.

I use the ECLS-B to estimate distinct CES skill accumulation technologies for couples and single mothers. I chose to use the ECLS-B rather than the PSID CDS because of its large sample and its emphasis on the time investments of both mothers and fathers in their child, and it contains more observations at earlier ages and for a larger sample than that available in the PSID CDS. One drawback of the ECLS-B relative to the PSID CDS is that it does not have a time diary: rather, parents report frequency of activities they do with their child on a daily, weekly, or monthly basis. I use an imputation strategy, described in detail in chapter 3 of this dissertation, so map from frequencies to levels of time inputs from parents in the ECLS-B.

1.3 Empirical Evaluations of Child Care Subsidies

Chapters 2 and 3 of this dissertation lay out a model and estimation for an environment in which the government subsidizes non parental child care. In practice, treatment effects for subsidized child care during early childhood are estimated from randomized control trials (RCTs) like the Abecedarian or Perry Preschool Projects, or from natural experiments using large-scale policy reforms.

Treatment effects of subsidized child care from RCTs are limited by the non-representativeness of the target group: these RCTs were usually targeted at children from impoverished families, who are often single mothers. Treatment effects estimated from large-scale interventions, however, are limited by the need to find a natural experiment (through geographic variation in the intensity of the subsidy, for example), and by the quality of long-term measures of outcomes due to short panels (which forces authors to use childhood test scores or grade repetition, rather than adult earnings of the treated vs. untreated children). On the other hand, one benefit of large-scale policies it that they are often universal, so that (unlike the RCTs) the treatment effects of these interventions can be estimated across the entire income distribution for the children's families.

It is important to keep in mind that direct comparison of treatment effects from large-scale subsidies with those estimated from the RCT data is problematic, because the subsidized child care is not the same quality. That is, the high quality of the Abecedarian and Perry Preschool Programs is not replicated in large-scale subsidized child care at a national scale. Whether or not high-quality child care programs can be scaled up to reach large numbers of children with the same degree of effectiveness is an open question.

The main findings of the empirical evaluations can be summarized as being positive for the RCTs and mixed for the large-scale policies. Recently, both literatures have begun to emphasize heterogeneity in treatment effects, by gender of the child and the outside option for child care in the RCT literature, and by family income in the large-scale policy evaluations.

Specifically, evaluations of randomized control trials like the Abecedarian or Perry Preschool Projects, have found that the sign of the treatment effect for subsidized child care depends on what the subsidy is crowding out, such as home care versus alternative child care. This outside option is in turn predicted by family structure and income (Garcia, Heckman, and Ziff (2017), Anderson (2008)). Outcomes in these datasets are measured in a variety of ways and are available over long panels.

By comparison, studies of universal child care subsidies examine Quebec (Kottelenberg and Lehrer (2017), Baker, Gruber, and Milligan (2008), Baker (2011)), Norway (Havnes and Mogstad (2014)), Denmark (Gupta and Simonsen (2010)), and Argentina (Berlinski, Galiani, and Gertler (2009)). These studies measure child outcomes in various ways, depending on the quality of available data: some use child test scores, while others use actual earnings data for the children as adults. Keeping this caveat in mind, the literature has found mixed effects on child outcomes. For example, Baker, Gruber, and Milligan (2008) find negative average treatment effects, while Berlinski, Galiani, and Gertler (2009) find positive ones. Recent papers which focus on heterogeneity in treatment effects include Havnes and Mogstad (2014) and Kottelenberg and Lehrer (2017). They find that children from poor families benefit the most, while those from families with higher incomes actually seem to be worse off due to the child care subsidies.

The fact that both Havnes and Mogstad (2014) and Kottelenberg and Lehrer (2017) find negative treatment effects for children in the upper parts of the family income distribution is puzzling. If child care is an input into skill production for children, and child care subsidies lower the cost of that input, then these subsidies weakly expand a families' choice set. That is, enrollment is not compulsory, and yet some families enroll children in child care programs which adversely affect those children's outcomes. From a decision-theory point of view, there is no reason for parents to rationally do this if they care about their children's outcomes. This is why general equilibrium, rather than partial equilibrium, is the appropriate framework to examine the long-run effects of child care subsidies.

Child care subsidies are of course not the only policy instrument that affects parental investment in child skill (Mullins (2018), Low, Meghir, Pistaferri, and Voena (2017), Guner, Kaygusuz, and Ventura (2016)). Although there is merit in pointing out the perhaps unintended effects of other policies on child skill, I chose child care subsidies to emphasize clarity of interpretation in my policy analysis.

Chapter 2

Child Care Subsidies with One- and Two-Parent Families

2.1 Introduction

Large-scale subsidized child care programs have been implemented around the world.[1] These large-scale programs are built on the success of smaller programs, which provided enriched early childhood environments for children from poor families and have shown potentially major gains from public intervention.[2] Empirical evaluations of large-scale child care subsidies have uncovered heterogeneous treatment effects along the dimensions of family income, family structure, and child gender, with conclusions differing by the country or state whose program is being evaluated.[3] Designs of these programs vary, but eligibility is usually universal or means-tested (targeted to poor families).

In this paper, I pursue a structural macroeconomic approach to analyze large-scale child care subsidies in the United States. The approach allows me to transcend local confounding factors, acknowledge general equilibrium effects on wages, the government budget, and marriage, and to evaluate policy designs that have not been already implemented. Specifically, I examine child care subsidies under three eligibility rules: universal, subsidies to the poor, and subsidies to one-parent families. To do this, I construct a general equilibrium overlapping generations model with families that are heterogeneous in income and marital status. My model takes into account endogenous family formation and altruistic parental

[1] Examples include: Oklahoma (started in 1998), Quebec (1997), Argentina (1993), Norway (1975), and Denmark (1964).

[2] Two well-known examples are the Abecedarian Program and the Perry Preschool Program. See Anderson (2008), Garcia, Heckman, Leaf, and Prados (2016), and Baker (2011).

[3] Studies of large-scale child care subsidies include Baker, Gruber, and Milligan (2008), Gupta and Simonsen (2010), Berlinski, Galiani, and Gertler (2009), Havnes and Mogstad (2014), Kottelenberg and Lehrer (2017), Blau and Currie (2006).

investment in child skill (Becker and Tomes (1979)). The three eligibility rules I examine permit a meaningful comparison between a targeting rule a model like mine can evaluate and more commonly implemented policies. In my analysis, I focus on the distribution of welfare gains across gender, skill, marital status, and age.

Comparing these subsidies in general equilibrium allows me to incorporate adjustments in labor income taxes, wages, and marriage decisions in response to policy. This feedback can magnify or mitigate the gains from the subsidy. For example, increases in skill, if accompanied by much higher labor income taxes to fund those increases, may not raise disposable earnings, consumption, or welfare for some groups. This is the case under a subsidy targeted to the poor. Similarly, endogenizing wages allows changes in the aggregate stock of skill to affect its rate of return, while endogenizing the family formation decision allows policy to affect the mass and composition of one- and two-parent families raising children in the economy. In fact, I find that the marriage rate decreases under the two targeted subsidies I consider, but increases under a universal child care subsidy. The family formation channel in my model is similar to that of Abbott, Gallipolli, Meghir, and Violante (2018), who examines how college tuition subsidies can affect the composition of marriages in the economy, but who do not allow for single-parent families to exist in equilibrium. The baseline framework into which I incorporate and endogenize heterogeneous family structures has been used to analyze the interaction of policy and skill investment in many studies, including Restuccia and Urrutia (2004), Lochner and Monge-Naranjo (2011), Guner, Kaygusuz, and Ventura (2016), Gayle, Limor, and Soytas (2017), Daruich (2017), Caucutt and Lochner (2017), Abbott, Gallipolli, Meghir, and Violante (2018), and Lee and Seshadri (2018).

The subsidies I evaluate generate welfare gains by partially addressing the market failures of the environment I construct. This model has four main sources of welfare gains: it insures against a low initial skill for the child, against being born into a family with fewer resources, and against a poor outcome in a frictional marriage market. In addition, it partially addresses a fiscal externality to parental investments in their child's skill.

To be specific, all newborns face risk over their initial skill, because a low draw makes good outcomes in adulthood harder to achieve. Similarly, the risk of being born into a low-resource family matters because parents and children cannot contract with one another (Cunha and Heckman (2007)). The resources available to invest in the child are therefore determined by the parent's permanent income, not the child's permanent income. The government, however, can use child care subsidies to lower the price of parental investment in children's skill, and then tax the child's labor earnings when they are older. In this way a child care subsidy combined with a labor income tax mimics a contract the child and parent would like to make with each other. With regard to marriage market risk, child care subsidies can partly insure individuals against parenting alone or with a low-skill spouse by lowering the costs of being a parent. Finally, the fiscal externality arises from an endogenous labor income tax. Taxpayers cannot use individual contracts to encourage others to invest in their children, so that the tax base expands and the labor income tax can decrease for everyone. A child care subsidy partially addresses this missing market by lowering the cost of investment in child skill, using funds contributed from everyone in the economy.

In my environment, one- and two-parent families invest in their children using distinct technologies. Because there is no existing estimation appropriate for my specification, I estimate these functions using a nationally representative panel dataset from the US Department of Education—the Early Childhood Longitudinal Study, Birth Cohort (ECLS-B). The ECLS-B is designed to be representative of the entire population of parents in the United States with 9-month-old children in 2001, and follows over 10,000 children from 9 months to kindergarten entry, recording their skill at 9 months, 2 years, 4 years, and 5 years of age. In addition to providing a rich source of information on the evolution of skill in young children, this survey is unique in its emphasis on the role of fathers in child development. For almost 1,000 couples over the first three waves of this survey, one can observe the couple's hourly wages, their distinct time investments in their children, and their use of non-parental child care as well as the child care price. For families with single mothers, the analogous sample contains about 500 families.

The panel nature of the ECLS-B allows me to control for fixed effects within a family when implementing the estimation of skill accumulation parameters. This accounts for unobserved and time-invariant heterogeneity in parenting productivity, child attributes, and local environment that affect the relative productivites of parental time and child care for investment in skill. My model specification for two-parent families is related to the skill accumulation technology in Del Boca, Flinn, and Wiswall (2014), who use the Panel Study of Income Dynamics, Child Development Supplement (PSID CDS) to estimate a Cobb-Douglas technology for couples with one or two children with separate time inputs from the mother and the father. Like them, I find that fathers play a non-negligible role in the raising of young children. I provide a second estimation for single mothers, which allows a new comparison of the ways that one- and two-parent families interact with non-parental care. I find that the input composition of investments is not very sensitive to price changes, and that single mothers rely more on non-parental child care for investment than two-parent families do. Consequently, although the price of investment is sensitive to the price of child care for both one- and two-parent families, the estimation indicates that this sensitivity is higher for one-parent families.

Using my framework to compare universal subsidies, subsidies to single mothers, and subsidies to poor families, I find that ex ante welfare for families in the baseline economy increases by 5.9%, with a 70% universal subsidy. Subsidies to single mothers can only reach a level of 2.4% welfare gains ex ante (at an 85% subsidy level), and poverty-tested subsidies provide a 2.0% gain (at an 85% subsidy level). Universal subsidies yield the highest welfare gains because they more fully insure newborns against the risks they face (which include both their initial skill and their family), without requiring an increase in the labor income tax in order to fund them. Subsidies to the poor disincentivize skill investment and require an increase in the labor income tax to balance the government budget; subsidies to single mothers, although they do not disincentivize skill investment, provide less insurance than universal subsidies.

This paper provides three main contributions. First, I incorporate both one- and two-parent families into an overlapping generations framework to analyze subsidies to investment in children's skill. Second, parallel to this heterogeneity in family structures, I introduce heterogeneity in the technologies that parents use to invest in their children, and provide estimates of these technologies. Finally, I allow the population to endogenously sort into the two structures via a marriage market, so that policy can affect the mass and composition of parents in one- and two-parent families.

The rest of the paper proceeds as follows. In section 2, I lay out the model. Section 3 presents the model parameterization, including the estimation of the skill accumulation technologies. Section 4 reports the subsidy design and level that maximizes ex ante welfare and interprets the distribution of welfare gains under each eligibility rule. Section 5 concludes.

2.2 The Model

There are four sets of agents in the economy: consumers, a representative firm, the government, and a non-parental child care provider. Consumers are grouped into families with either one or two parents, who altruistically invest in their child's skill with their own time and purchased child care time, as well as choosing consumption, savings, labor supply and leisure. The way time inputs affect children's skill is determined by a skill accumulation technology, which is indexed by the number of parents in the household. Given prices for labor and capital, the firm chooses labor and capital inputs to maximize profits subject to a free-entry condition. The government chooses labor income taxes to finance lump-sum transfers and non-parental child care subsidies. A child support system exists, enforced by the government, where single fathers contribute a lump-sum amount that is redistributed lump-sum and equally to all single mothers. Finally, the non-parental child care sector supplies child care at the amount demanded in equilbrium, at a price equal to some fraction of the average hourly wage.

The Life Cycle of Consumers

Each individual lives for four periods of equal length: childhood, parenthood, adult worker, and old worker. During childhood, an individual makes no decisions: she is a passive recipient of consumption and investment chosen by her family. Upon independence, the individual leaves with her skill to start the parenting phase as an independent decision maker.

At the beginning of the parenting phase, before any decisions are made, everyone participates in a marriage market. This market is modelled as a random search with an arrival rate of one: a potential match is drawn from the skill distribution of the other gender in the same generation. Once assigned a potential spouse, and knowing that fertility is exogenous and certain in the environment, the agent compares the expected present discount value of parenting alone or in a couple.

The gains from joining a couple are reflected by higher efficiencies in translating income into consumption (introduced with consumption equivalence scales) while the costs are reflected by the fact that spouses must compromise on time use. In addition, couples use the two-parent technology to invest in their children. Being a single parent, meanwhile, is an outside option to marriage that differs by gender. For a woman, single parenthood means that she keeps her children with her, using the one-parent technology to invest in them, and receives lump-sum child support transfers from single fathers. For a man, single parenthood means that he cannot directly affect his child's skill with his time use, but does have to pay a lump-sum child support tax to the single mother.[4]

Whether parenting alone or in a couple, the lifetime utility of any individual contains a term that incorporates rational expectations about the lifetime utility of one's child.[5] This expectation is taken over the initial skill of the child, which is drawn from an exogenous distribution and is unknown when the marriage decision is made. The predictive power of a child's initial skill for lifetime utility, however, endogenously responds to policy. This, along

[4]The outside option to parenting in a couple differs by gender in the model because, empirically, the vast majority of single parents who are raising young children in their home are women. For a discussion of what the ECLS-B offers in terms of discipline on contributions of parental time from single fathers, see the appendix.

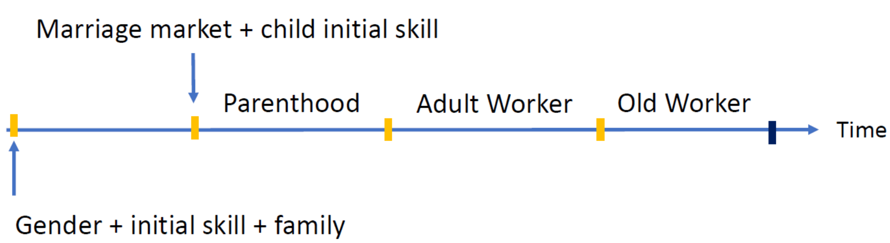
[5]This is what makes parents altruistic. An alternative way of incentivizing intergenerational transfers is through paternalistic preferences, or "warm glow" returns (Andreoni (1990)). The benefit of motivating parents with altruism is that the returns to investment can respond rationally and endogenously to policy, because parents fully incorporate the economic returns to their investment in terms of their child's lifetime utility. The main benefit of a paternalistic specification is its tractability and flexibility in matching parenting behaviors. Some models combine the two, and include both altruism and a paternalistic preference for, say, college attainment which is distinct from its monetary returns. For an application of paternalistic preferences to intergenerational transfers of wealth, see De Nardi (2004).

with the endogenous labor income tax, is the channel by which policy affects the family formation decision.

Once a potential couple has compared their two alternatives, a marriage requires that both the husband and wife accept the match (the two individuals remain single otherwise). After the marriage market, single mothers and couples draw the initial skill of their two children, which is the same for both children (single mothers and couples each raise both a son and a daughter). Given their beliefs about how skill affects lifetime utility, parents then choose the level and composition of investment in their children's skills. At the end of the parenting phase, the children leave their parents' state space, and parents enjoy an altruistic return from the lifetime utilities of their children.

The problems of the adult worker and the old worker periods differ only in that old workers die at the end of their phase, so in the last period of the old worker phase there is no savings decision. Before old age there are no borrowing constraints: borrowing constraints on the parents are not a source of market incompleteness in this model. To summarize, the shocks in the life of an individual are their own initial skill, their gender, the family that raises them, their potential spouse, and the draw of their children's initial skill. Figure 1 illustrates the timing of the phases and the draws of these shocks.

Figure 2.1: Life Cycle of the Consumer



One- and Two-Parent Families

The two family structures are different in that they have different efficiencies of consumption ($\{\phi_s\}$ and $\{\phi_{mc,p}, \phi_{mc,w}\}$, which are consumption equivalence scales) and are allowed to have different marginal utilities of leisure (which will be reflected in the parameters of the period utility functions (u_s, u_{mc}), whose functional forms are defined in the model parameterization section). Married couples compromise on their leisure decision in the sense that it has to be the same for both members of the couple. They also use a skill accumulation technology specific to two-parent families to invest in their children (f_{mc}). Similarly, single mothers invest in children using a skill technology specific to them (f_s).

Family Problems

The solution to a family's life-cycle problem is a set of policy functions and value functions. These functions are defined for young adults (before the marriage decision) with skill θ and gender $g \in [1, 2]$ where 1 is father and 2 is mother, and for one and two-parent families at each period $j \in \{1, 2, 3\}$ (with the numbers corresponding to parenthood, adult worker, and old worker, respectively), state z, and the skill of the single parent θ or the skills of the father f and mother m of the couple (θ_f, θ_m). The state space z always contains assets, which are zero at the start of life, and during parenthood also contains the initial skill of the child. The value functions are:

- For young adults: $V_0(\theta, g)$
- For single mothers: $V_j^{SM}(z; \theta)$
- For single fathers: $V_j^{SF}(z; \theta)$
- For married couples: $V_j^{MC}(z; \theta_f, \theta_m)$

The Marriage Decision and Probability of Getting Married

The policy function that maps from the type of the spouse to a yes or no marriage market decision ($d_{mm}(\theta_f, \theta_m, g) \in \{0, 1\}$, given the skill θ_m or θ_f and gender g of the decision maker), is a threshold strategy in the skill of the potential spouse. It solves:

$$
\begin{aligned}
d_{mm}(\theta_f, \theta_m, 1) &= \arg\max_{\delta \in \{0,1\}} \left[\delta \int_{\theta_c} V_1^{MC}(z; \theta_f, \theta_m) \pi(\theta_c) d\theta_c + (1 - \delta) \int_{\theta_c} V_1^{SF}(z; \theta_f) \pi(\theta_c) d\theta_c \right] \quad (2.1) \\
d_{mm}(\theta_f, \theta_m, 2) &= \arg\max_{\delta \in \{0,1\}} \left[\delta \int_{\theta_c} V_1^{MC}(z; \theta_f, \theta_m) \pi(\theta_c) d\theta_c + (1 - \delta) \int_{\theta_c} V_1^{SM}(z; \theta_m) \pi(\theta_c) d\theta_c \right]
\end{aligned}
$$

where $z = \{0, \theta_c\}$ is the state space in the first period of life which reflects the zero wealth all consumers start with and the initial skill of the child θ_c, and the distribution of initial child skills has a probability density function given by $\pi(\theta_c)$.

Draws for Marriage Offer, Partner Type, and Child Skill

The expected lifetime utility of a child enters into the parent problem. It is an expectation over outcomes in the marriage market and draws of initial child skill when they become a parent, as a function of their skill and gender g:

$$
\begin{aligned}
V_0(\theta_f, 1) &= \int_{\theta_m} \left[\mathbb{I}_d \int_{\theta_c} V_1^{MC}(z; \theta_f, \theta_m) \pi(\theta_c) d\theta_c + (1 - \mathbb{I}_d) \int_{\theta_c} V_1^{SF}(z; \theta_f) \pi(\theta_c) d\theta_c \right] \mu(\theta_m, 2) d\theta_m \quad (2.2) \\
V_0(\theta_m, 2) &= \int_{\theta_f} \left[\mathbb{I}_d \int_{\theta_c} V_1^{MC}(z; \theta_f, \theta_m) \pi(\theta_c) d\theta_c + (1 - \mathbb{I}_d) \int_{\theta_c} V_1^{SM}(z; \theta_m) \pi(\theta_c) d\theta_c \right] \mu(\theta_f, 1) d\theta_f
\end{aligned}
$$

where $\mathbb{I}_d \equiv d_{mm}(\theta_f, \theta_m, 1) \times d_{mm}(\theta_f, \theta_m, 2)$ indicates a mutual acceptance of the match, $z = \{0, \theta_c\}$ and θ_c is the child's initial skill, drawn independently from an exogenous distribution. In addition, $\mu(\theta, g)$ is the endogenous distribution over adult skill θ and gender g. In the following subsections, I define the life-cycle problem for each of the three family types: single mother, single father, and married parents. In all the family problems, τ_y is the labor income tax, τ_n is the child care subsidy, T are lump-sum transfers, and T_{cs} are child support payments.

Single Mother Problem

During parenthood, a single parent chooses consumption c, savings $a^{'}$, non-parental child care time n, her own time investments in her child q, and leisure ℓ to solve the following problem:

$$
\begin{aligned}
V_1^{SM}(a, \theta_c; \theta) &= \max_{c, a^{'}, n, q, \ell} u_s\left(\frac{c}{\phi_s}, \ell\right) + \beta V_2^{SM}\left(a^{'}; \theta\right) + b \sum_g V_0\left(\theta_c^{'}, g\right) \\
c + a^{'} + (1 - \tau_n) p_n n &\leq (1 + r) a + (1 - \tau_y) w \theta (1 - \ell - q) + T + T_{cs} \\
\ell, n, q \in [0, 1] & \qquad \ell + q \leq 1 \quad n + q \leq J_{child} \\
\theta_c^{'} &= f_s(\theta_c, n, \theta q)
\end{aligned}
\tag{2.3}
$$

Here, and in what follows, ϕ_s is the consumption-equivalence scale for single-parent families with children, β is the discount factor (patience), b is the altruism parameter, w denotes the wage rate, and p_n denotes the price of non-parental child care. J_{child} is the time endowment of the child. During the parenting phase, parental and non-parental child care time investments in child skill affect skill in the next period according to the single parent production technology. The final child skill enters the objective function of the mother through an altruism term $b \sum_g V_0\left(\theta_c^{'}, g\right)$, which weights the expected lifetime utility of the child $V_0\left(\theta_c^{'}, g\right)$ with the altruism coefficient b. Throughout their childhood, the mother cannot direct investments separately to the son and daughter. Her children are born with the same skill and receive the same investments. For a regression motivating this assumption, see the appendix.

During the adult worker phase, the single mother consumes c, saves $a^{'}$, and enjoys leisure ℓ:

$$
\begin{aligned}
V_2^{SM}(a; \theta) &= \max_{c, a^{'}, \ell} u_s(c, \ell) + \beta V_3^{SM}\left(a^{'}; \theta\right) \\
c + a^{'} &\leq (1 + r) a + (1 - \tau_y) w \theta (1 - \ell) + T \\
\ell &\in [0, 1]
\end{aligned}
\tag{2.4}
$$

During the old worker phase, the single mother consumes c, and enjoys leisure ℓ. She dies with zero savings:

$$\begin{aligned} V_3^{SM}\left(a;\theta\right) &= \max_{c,\ell} u_s\left(c,\ell\right) \\ c &\leq (1+r)\,a + (1-\tau_y)w\theta\left(1-\ell\right) + T \\ \ell &\in [0,1] \end{aligned} \tag{2.5}$$

Single Father Problem

During parenthood, a single father chooses consumption c, savings $a^{'}$, and leisure ℓ to solve the following problem. They also have to pay child support T_{cs} which is received by single mothers:

$$\begin{aligned} V_1^{SF}\left(a;\theta\right) &= \max_{a^{'},c,\ell} u_s\left(c,\ell\right) + \beta V_2^{SF}\left(a^{'};\theta\right) + \zeta\left(\mu\right) \\ c + a^{'} + T_{cs} &\leq (1+r)\,a + (1-\tau_y)w\theta\left(1-\ell\right) + T \\ \ell &\in [0,1] \end{aligned} \tag{2.6}$$

Single fathers do not internalize their child's outcome directly; rather, their payoff includes some function of the aggregate endogenous state $\zeta\left(\mu\right)$, which I will specificy in the parameterization section. This specification allows policy to affect single fathers through the distribution of skill μ. This specification also does not allow a single father to use his own time to invest in his children. I start from the assumption that resident fathers are more able to invest in their children's skill with their own time than non-resident fathers. Here, I take that assumption to an extreme and abstract from single father time investments entirely, as well breaking the link between the objective function of the single father and the specific outcomes of their children.

During the adult worker phase, single fathers consume c , save $a^{'}$, and enjoy leisure ℓ by solving the following problem:

$$\begin{aligned} V_2^{SF}\left(a;\theta\right) &= \max_{c,a^{'},\ell} u_s\left(c,\ell\right) + \beta V_3^{SF}\left(a^{'};\theta\right) \\ c + a^{'} &= (1+r)\,a + (1-\tau_y)w\theta\left(1-\ell\right) + T \\ & \quad \ell \in [0,1] \end{aligned} \tag{2.7}$$

During the old worker phase, the single father consumes c and enjoys leisure ℓ. He dies with zero savings:

$$\begin{aligned} V_3^{SF}\left(a;\theta\right) &= \max_{c,\ell} u_s\left(c,\ell\right) \\ c &\leq (1+r)\,a + (1-\tau_y)w\theta\left(1-\ell\right) + T \\ \ell &\in [0,1] \end{aligned} \tag{2.8}$$

Married Couple Problem

During the parenting phase, married couples choose consumption c, savings $a^{'}$, leisure ℓ, non-parental child care time n and parental time inputs $\{q_f, q_m\}$. Like single mothers, married couples cannot direct investment by child gender and enjoy an altruistic return from the lifetime utility of their children as a function of their skill at adulthood.

$$\begin{aligned}
V_1^{MC}(a, \theta_c; \theta_f, \theta_m) &= \max_{c,a^{'},\ell,n,q_f,q_m} u_{mc}\left(\frac{c}{\phi_{mc,p}}, \ell\right) + \beta V_2^{MC}\left(a^{'}, \theta_c^{'}; \theta_f, \theta_m\right) + b\sum_{g_{child}} V_0(\theta_c, g) \\
c + a^{'} + (1-\tau_n)\, p_n n &\leq (1+r)\, a + (1-\tau_y) w \left[\theta_f(1-\ell-q_f) + \theta_m(1-\ell-q_m)\right] + T \qquad (2.9) \\
\ell, n, q_f, q_m \in [0,1] & \quad \ell + q_f \leq 1 \quad \ell + q_m \leq 1 \quad n + q_f + q_m \leq J_{child} \\
\theta_c^{'} &= f_{mc}(\theta_c, n, \theta_f q_f, \theta_m q_m)
\end{aligned}$$

Here, $\phi_{mc,p}$ is the consumption-equivalence scale of a family with two adults and two children. The specification I use for the married couple problem is based on Guvenen and Rendall (2015). As in that study, I motivate the perfect complementarity in leisure of the spouses with time use data as documented in Aguiar and Hurst (2007), Table V. However, unlike Guvenen and Rendall (2015), I do not make the marginal utility of leisure stochastic. In my model, by comparison, the only source of risk in marriage is the initial skill of the children the couple has together.

During the adult worker phase, married couples consume c, save $a^{'}$, and enjoy leisure ℓ:

$$\begin{aligned}
V_2^{MC}(a; \theta_f, \theta_m) &= \max_{c,a^{'},\ell} u_{mc}\left(\frac{c}{\phi_{mc,w}}, \ell\right) + \beta V_3^{MC}\left(a^{'}; \theta_f, \theta_m\right) \qquad (2.10) \\
c + a^{'} &\leq (1+r)\, a + (1-\tau_y) w \left[\theta_f(1-\ell-q_f) + \theta_m(1-\ell-q_m)\right] + T \\
\ell &\in [0,1]
\end{aligned}$$

Here, $\phi_{mc,w}$ is the consumption-equivalence scale of a family with two adults and two children. During the old worker phase married couples consume c and enjoys leisure ℓ. They die with zero savings:

$$\begin{aligned}
V_3^{MC}(a; \theta_f, \theta_m) &= \max_{c,\ell} u_{mc}\left(\frac{c}{\phi_{mc,w}}, \ell\right) \qquad (2.11) \\
c &\leq (1+r)\, a + (1-\tau_y) w \left[\theta_f(1-\ell-q_f) + \theta_m(1-\ell-q_m)\right] + T \\
\ell &\in [0,1]
\end{aligned}$$

Government

The government collects revenue from labor income taxes τ_y to finance lump-sum transfers T and non-parental child care subsidies τ_n. The variable H is the aggregate supply of labor efficiency units, and N is the aggregate demand for non-parental child care:

$$\tau_y w H = T + \tau_n p_n N \tag{2.12}$$

Representative Firm

The firm chooses capital K_F and labor inputs H_F to maximize profits, taking prices r and w as given. The parameter δ_F is the depreciation rate of capital.

$$\max_{K,H} \quad \left\{ K_F^{\alpha_F} H_F^{1-\alpha_F} - wH_F - (1 + r - \delta_F) K_F \right\} \tag{2.13}$$

Non-parental Care Sector

The non-parental child care sector provides N units of non-parental child care at price p_n. The price of non-parental child care is set as a constant fraction κ of the average earnings per unit of time:

$$p_n = \kappa \sum_g \int_\theta w\theta\mu(\theta, g)\, d\theta \tag{2.14}$$

This allows the price of non-parental child care to adjust with the average level of skill in the economy, but without specifying a production function for non-parental child care.

Equilibrium

Given a government policy $\{\tau_n\}$, transfers T, and child support T_{cs}, a stationary equilibrium is defined as:

- Prices $\{r, w\}$ and tax τ_y,
- Individual marriage decision rules for each type and potential spouse,
- Policy functions for each family type for each period of life j and state z,
- Value functions at the beginning of adulthood V_0 and for each family type V^{SM}, V^{SF}, V^{MC},
- A non-parental child care price p_n,
- Joint distribution μ over adult skill and gender,

such that:

- Capital and labor markets clear.
- The government balances its budget taking prices as given.

- Family decision rules solve their dynamic problems taking prices and taxes as given.
- The non-parental child care price p_n is a constant fraction κ of average hourly earnings.
- The joint distribution μ is stationary.

Market Incompleteness in the Environment

As I discuss in the introduction, the subsidies I evaluate generate welfare gains by partially addressing the market failures of the environment I construct. A child care subsidy in this environment yields welfare gains from four main sources: it insures against a low initial skill for the child, against being born into a family with fewer resources, and against a poor outcome in a frictional marriage market. In addition, it partially addresses a fiscal externality to parental investments in their child's skill.

To be specific, all newborns face risk over their initial skill, because a low draw makes good outcomes in adulthood harder to achieve. Similarly, the risk of being born into a low-resource family matters because parents and and children cannot contract with one another (Cunha and Heckman (2007)). The resources available to invest in the child are therefore determined by the parent's permanent income, not the child's permanent income. The government, however, can use child care subsidies to lower the price of parental investment in children's skill, and then tax the child's labor earnings when they are older. In this way a child care subsidy combined with a labor income tax mimics a contract the child and parent would like to make with each other. With regard to marriage market risk, child care subsidies can partly insure individuals against parenting alone or with a low-skill spouse by lowering the costs of being a parent. Finally, the fiscal externality arises from an endogenous labor income tax. Taxpayers cannot use individual contracts to encourage others to invest in their children, so that the tax base expands and the labor income tax can decrease for everyone. A child care subsidy partially addresses this missing market by lowering the cost of investment in child skill, using funds contributed from everyone in the economy.

When I analyze the results of the policy experiment I conduct, I discuss how different specifications would affect the magnitude of welfare gains. In particular, I find that universal subsidies yield the highest welfare gains. This is in part because in this environment, there is risk over the initial skill of the child which is distinct from the equilibrium distribution of families and unaffected by policy (because the distribution of initial skill is exogenous). This makes a universal subsidy yield higher welfare gains than it would in an environment where everyone was born with the same initial skill or where the distribution of initial skill was endogenous.

2.3 Model Parameterization

To implement this model, I grouped the model parameters into those chosen externally, those estimated outside the model, and those calibrated inside the model. Parameters chosen externally are drawn from the literature, or using a common rule. Parameters estimated outside the model refers to the set of parameters that I discipline with data, but for which I do not have to solve for the model equilibrium to check how the model moment compares with the moment in the data. These include the skill accumulation technology parameters, among others, which are estimated using first-order conditions of the parenting problems (I will explain this estimation in more detail further on in this section). Finally, the set of parameters calibrated inside the model refers to those parameters for which I have to solve for the model equilibrium in order to generate a model moment to compare with the data. These parameters include the altruism coefficient b, the productivity of investment λ, and the marginal utility of leisure for singles (ψ_s) and couples (ψ_{mc}).

2.3.1 Functional Form Assumptions

Period Utility Functions

Utility functions are defined separately for one- and two-parent families:

$$u_{type}(c, \ell) = \log(c) + \psi_{type} \log(\ell), \quad type \in \{s, mc\} \tag{2.15}$$

Single Father Altruism Term

I set the single father term $\zeta(\mu)$ equal to the minimum possible outcome in the economy:

$$\zeta(\mu) = \min_{\theta, g} V_0(\theta, g) \tag{2.16}$$

In practice, reasonable values for child support transfers (see appendix) are not sufficient to reach the targeted marriage rate in the internal calibration without further penalizing the single father path. With this specification, I still make the single father lifetime utility depend directly on the skill distribution in the economy, as with the other parenting problems. By comparison, using the average outcome does not sufficiently penalize the single father.

Distribution of Initial Child Skill

I assume that initial child skill is drawn independently and identically from $\pi(\theta_c)$, which I set as a uniform distribution. For motivating regressions for the i.i.d. assumption, see the appendix.

Skill Accumulation Technologies

I specify the skill accumulation technologies to be nested constant elasticity of substitution (CES) functions with inputs of non-parental time, mother time, and father time (which aggregate to investment I_{type}) and between investment and the current stock of skill (which aggregate to tomorrow's skill). Specifically, the functional forms of the dynamic equation for skill accumulation, and how investment is generated from time inputs for the two family structures, are as follows:

$$\begin{aligned} f_{type}(\cdot) &= \left[\upsilon\left(\lambda I_{type}\right)^{\xi} + (1-\upsilon)\,\theta_c^{\xi}\right]^{\frac{1}{\xi}}, \quad type \in \{s, mc\} && (2.17)\\ I_s &= \left[\gamma_s\left(\theta_m q_m\right)^{\eta_s} + (1-\gamma_s)\,(n)^{\eta_s}\right]^{\frac{1}{\eta_s}} && (2.18)\\ I_{mc} &= \left[(1-\alpha)\left(\gamma\left(\theta_m q_m\right)^{\eta} + (1-\gamma)\,(n)^{\eta}\right)^{\frac{\rho}{\eta}} + \alpha\left(\theta_f q_f\right)^{\rho}\right]^{\frac{1}{\rho}} && (2.19) \end{aligned}$$

The parameters γ, γ_s and α control the relative level of inputs given a price ratio. The values of η, η_s, and ρ control the percentage change in the ratio of inputs for a percentage change in the ratio of their prices. With this parameterization, I allow for mother time and non-parental child care time to interact differently across household structures (i.e., I do not impose $\gamma = \gamma_s$ and $\eta = \eta_s$). [6]

In Chapter 3 of this dissertation, I explain in detail the data and methods used to estimate the parameters of this skill accumulation technology. Here, I simply report the final point estimates in Table 2.1.

Table 2.1: Parameters of the Human Capital Accumulation Technology

ρ	α	η	γ	η_s	γ_s	ξ	υ
-1.86	0.07	-1.01	0.24	-2.25	0.03	-1.2	0.82
(0.42)	(0.03)	(0.21)	(0.05)	(0.90)	(0.04)	(0.14)	(0.03)

Note: Standard errors are in parentheses, clustered at the level of state of residence and calculated using the delta method.

[6] A different and common way of specifying the investment aggregator is to include money (goods) and parental time inputs. See the appendix for an exercise where I measure the contribution of child care costs to expenditures on children in different age groups, using the 2001 PSID and the 2002 PSID CDS. I find that child care costs are a sizeable compoenent of money spent on children by any measure of spending on children I consider (the share ranges from 50 to 70 percent of total spending). My specification makes explicit how expenditures on children affect child skill accumulation through child time use.

2.3.2 Externally Chosen Parameters

The length of a lifetime and of each phase are proportional to 20 years of childhood and 60 years of adulthood (death at age 80). The discount (patience) factor is set to a yearly value of 0.96 to match the risk-free interest rate. The time endowment for early childhood is 5 years, which is 0.25 the length of a period (20 years). The capital share of the production technology is set to a standard value of 0.33, and the depreciation rate of capital is chosen to be 0. Finally, the consumption equivalence (CE) scales are set using the 1994 scales from the Organisation for Economic Co-operation and Development (OECD). These scales assign a value of 1 for the first adult, and 0.5 for the subsequent adults; for each dependent the weight is 0.3. They adjust money spent on consumption into units of consumption for each member of the household. Once children leave the family, the equivalence scale for single mothers goes back to 1, and the scale for couples falls to 1.5. This is summarized in Table 2.2.

Table 2.2: Externally Chosen Parameters

Symbol	Name	Value
β	Patience	0.96^{20}
J_{child}	Duration Early Childhood	0.25
$\{\alpha_F, \delta_F\}$	Production Technology	$\{0.33, 0\}$
$\{\phi_s\}$	OECD CE Scales: 1 adult, 2 children	$\{1.6\}$
$\{\phi_{mc,p}, \phi_{mc,w}\}$	OECD CE Scales: 2 adult, 2 children	$\{2.1, 1.5\}$

2.3.3 Other Externally Estimated Parameters

In Table 2.3, the level of lump-sum transfers T, the level of child support payments T_{cs}, and the price of p_n are set to 8% of output, 28% of the average per-family transfer, and 35% of the average mother's wage, respectively. The first target is from the ratio of government transfers to persons for federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, refundable tax credits, and other (which includes payments to nonprofit instutitions and student loans, among other categories) to GDP from the National Income and Production Accounts (NIPA) tabulations.The second is the ratio of average child support payments per month per capita to average monthly government transfers per family. The third is the average ratio of hourly price of non-parental child care to hourly wages of mothers in the ECLS-B. See the appendix for further details on the estimation of these parameters.

Table 2.3: Externally Estimated Parameters

Symbol	Name	Source	Value
T	Transfers	NIPA	8% of output
T_{cs}	Child Support	Census and NIPA	28% of T
κ	p_n coefficient	ECLS-B	35% ave. mother's wage

2.3.4 Internally Calibrated Parameters

The parameters $b, \psi_s, \psi_{mc}, \lambda$ are chosen to bring the model moments in the baseline as close as possible to the moments in the "Data" column of Table 2.4. The coefficient b controls the degree of altruism; ψ_s, ψ_{mc} are the marginal utilities of leisure for singles and married couples, respectively. The parameter λ is a shifter in the skill technology that scales up investment into efficiency units in the production of skill.

The moments I chose these parameters to match in the model compared with the data are the correlation of child skill with family income, the average labor supply of parents with children under 5 who are between 20 and 80 years old, the percent of single mothers raising children under 5, and the average time invested by parents of each family structure type. Note that the correlation of child skill and family income grows to 0.33 by the time the child is 5 years old (ECLS-B). By contrast, when children are 9 months old, the measures of skill available in the data are uncorrelated with family income, although they do have predictive power for later child test scores at age 4. For regressions supporting these points, see the appendix.

There are two internal calibration moments for which it has proven difficult to match the model with the data: the correlation of child skill and family income and the levels of time investments. For the former, in this model the family incomes of couples and singles are quite different because the marriage market outcome is so closely related to skill. This means that the income distribution for families raising young children looks like two slightly overlapping distributions, one for single mothers and one for couples. Within each group the correlation of child skill and family income is lower than the statistic reported in Table 2.4; once the differences across family structures are accounted for the correlation looks much higher (compared to the correlation within family structures) because of the difference in income levels across the two structures is so high. For the latter, the levels of time investments are wrong in the sense that the magnitudes are off, but they are also in the wrong order: in the data, single mothers more education time, on average, than married fathers. In my model there is no gender wage gap, and the model lacks one reason why fathers in the data have a higher opportunity cost of their time than mothers do. Consequently, fathers in the model oversupply parenting time relative to single mothers.

Table 2.4: Internal Calibration: Targeted Moments + Parameters

Moment	Source	Data	Model
Corr(child skill, family income)	ECLS-B	0.33	0.39
Average labor supply	CPS	0.31	0.30
Percentage single mothers	ECLS-B	0.20	0.20
Average time investments	ECLS-B (MM,MF,SM)	(8,5.6,7)	(9.2,7.6,4.8)

Notes: Internal calibration targeted moments (data v. model).

Time investments are in units of hours per week.

Parameter	Name	Value
b	Altruism coefficient	$0.29 \times \beta$
$\{\psi_s, \psi_{mc}\}$	Marginal utility of leisure	$\{1.4, 1.4\}$
λ	Productivity of investment	45

Notes: Parameter names and values for internal calibration.

2.3.5 Untargeted Moments

In Table 2.5, I compare the implications of this model with six untargeted moments: the correlation of hourly wages in spouse (or cohabiting partners), the correlation of parental time inputs within a couple, the ratio of average hourly wages in married or cohabiting mothers with single mothers, the average ratio of mother and father time, the average ratio of mother time and non-parental child care time, and the marriage rate of the poor. For a comparison of the income distribution in the model with the data, see the appendix.

Table 2.5 shows that the random search marriage market I model captures an appropriate degree of assortative matching among spouses: the correlation of wages within couples is a close fit with the data. However, within a couple the model implies a higher correlation of time inputs than seen in the data. This partly reflects the fact that the levels of wages for men and women in the model economy have identical distributions, but in the data there is an unmodelled gender wage gap.

Table 2.5: Untargeted Moments (Model Fit)

Moment	Source	Data	Model
Correlation of wages within a couple	ECLS-B	0.28	0.29
Correlation of time inputs within a couple	ECLS-B	0.33	0.46
$\frac{\text{ave. hourly wages, single mothers}}{\text{ave. hourly wages, mc mothers}}$	ECLS-B	0.67	0.86
Average $\frac{\text{father time}}{\text{mother time}}$	ECLS-B	0.77	0.81
Average $\frac{\text{mother time}}{\text{non-parental child care time}}$	ECLS-B	0.37	0.18
Marriage rate of the poor	ECLS-B (wave 3)	52%	6.4%

Notes: Table 2.5 shows relevant moments for the implications of the model for sorting across family structures, assortative matching in marriage, patterns of time inputs for investments in child skill and the marital status composition of the poor. For the last moment, the poor in the model are defined as the bottom 20% of the pre-tax family income distribution corrected with CE scales. This follows the definition of poor families used in the policy exercise of the next section. Correlations are Pearson correlation coefficients, weighted with survey weights in the ECLS-B and using the analogous distribution in the model. Ratios use empirical moments from Tables 4 and 5 and analogous moments using the baseline equilibrium of the model.

To examine how mothers sort into the family structures, the third moment looks at the ratio of their wages across family structures. In the data, single mothers have lower hourly wages than married mothers. I assume that hourly wages are correlated with parenting productivity in my estimation, so the sorting across family structures has implications for the parenting productivities using each of the two technologies in the population. My model endogenously captures the qualitative attribute of the data: mothers with higher skill tend to parent in a couple. However, the average hourly wages of single and married mothers in the model are closer together than in the data. More higher-wage women parent as single mothers in the model than in the data.

The next two moments in Table 2.5 show the average ratios of time inputs in the model and in the data. The ratio of time inputs within a couple is quite close to the data, but the ratio of mother time with non-parental child care time is too low in the model. This reflects the fact that I did not model child care quality choice, so that only the intensive margin of non-parental child care time can adjust to increase child care's contribution to investment. Finally, the last row of Table 2.5 shows the marriage rate of the poor in the model compared with the data. The poor in the model are defined as the bottom 20% of the pre-tax family income distribution corrected with CE scales. This follows the definition of poor families used in the policy exercise of the next section. The close link between skill and marriage in this model means that poor parents (low earners) raising young children are rarely married.

2.4 Policy Experiment and Results

I vary τ_n over the interval $[0, 1]$ with three eligibility rules: universal, poverty-tested, and family-tested. Under a universal eligibility rule, every parent receives the subsidy, whereas under a poverty-tested rule only families in the bottom 20% of the CE-adjusted income distribution for parenting families in the baseline economy receive the subsidy. I chose this approach so that the mass of those eligible for the poverty-tested and single mother subsidy is the same in the baseline (20 percent of parenting families), which makes the two eligibility criteria more comparable. As Table 9 documents, in practice this means that in the baseline economy single mothers are overrepresented in the poor compared with the data. For each τ_n value I solve for equilibrium and record household decisions for every possible family type, as well as the endogenous population weights assigned to that family type.

A family type is the marriage structure (single mother, single father, or married couple), the productivity type of the parent(s), and the initial skill of the children (every single mother and married couple raises one boy and one girl). An equilibrium output in the model is structured like a survey dataset, with observations at the family type level and the endogenous population weights being analogous to survey weights. I use this information to compute welfare changes under different policies. For each eligibility criteria, I find the subsidy level that implements the largest ex ante welfare gain for children at birth. I compare these maximized welfare gains, looking at the distribution of these gains in the population after birth as well as across initial skill types of children before they know the family they will be raised in.

For the welfare criterion used to select the best eligibility rule, using either the baseline or the subsidized equilibrium distribution of families gives the same ranking. For decompositions of welfare changes, I use the distribution of families in the baseline equilibrium, and when I present changes in aggregates I use the distribution of families in the subsidized equilibrium. The former choice is to reflect how the distribution of families in the baseline (those a policymaker is answerable to when implementing these policies) will evaluate the proposed child care subsidy. The latter choice is to reflect how the subsidy's affect on individual choices in turn distorts aggregates in the subsidized equilibrium.

The rest of this section proceeds as follows. First, I report the formulas I use to calculate welfare gains for different levels and eligibility rules of the non-parental child care subsidy. Second, I report the best attainable outcome for each eligibility rule (universal, subsidies to the poor, and subsidies to single mothers), using ex ante welfare by child initial skill and decompositions of welfare gains by attributes of the population such as age, gender, marital status, and adult skill level. Differences between the effects of the three eligibility rules are apparent with the decompositions I report and analyze in the last subsection of these results. Third, I discuss how some features of my model specificaton are related to the results.

Welfare Measures

The equation to compute welfare gains for each type is:

$$\Delta W_j^{type}\left(z;\overrightarrow{\theta}|\tau_n\right) = \exp\left(\frac{V_j^{type}\left(z;\overrightarrow{\theta}|\tau_n\right) - V_j^{type}\left(z;\overrightarrow{\theta}|\tau_n = 0\right)}{\sum_{age=j}^{3}\left(\beta^{age-j}\right)}\right) \qquad (2.20)$$

where $type \in \{SM, SF, MC\}$ is the family structure type, z is the state space at that phase of life for that family type, and $\overrightarrow{\theta}$ is the vector of adult skill types, equal to θ in the case of single parents or θ_f, θ_m in the case of married couples. The state space z contains the asset stock; equation (2.20) uses the policy choice for that household at that age, in the previous period. The object $\Delta W_j^{type}\left(z;\overrightarrow{\theta}|\tau_n\right)$ is the consumption-equivalent change in lifetime utility evaluated at age j, for a given family structure and composition. To aggregate these welfare changes, I use the distribution of families in the subsidized or baseline equilibrium (I am specific in each case). I construct the distribution of families I use to aggregate welfare using the appropriate joint distribution μ, marriage decision rules, and the exogenous distribution from which the initial skill of the child is drawn.
Ex ante welfare changes in consumption-equivalent units are computed as follows:

$$\Delta W_0\left(\theta, g|\tau_n\right) = \mathbb{E}_{type,\overrightarrow{\theta},\theta_c}\left[\Delta W_1^{type}\left(z;\overrightarrow{\theta}|\tau_n\right)|\theta, g\right] \qquad (2.21)$$

This is the consumption-equivalent change in ex ante lifetime utility, evaluated at the skill type θ and gender g of the child, at an equilibrium indexed by the subsidy τ_n. Taking the expectation of ΔW_1^{type}, as defined in equation (2.20), over family structure *type*, the vector of skills for the adults in the family $\overrightarrow{\theta}$, and the initial skill of the child θ_c, requires the endogenous distribution of skill for the first two and the exogenous distribution of initial skill for θ_c. For the endogenous distribution of skill, one can use either the baseline or the subsidized equilibrium's distribution to calculate ex ante welfare gains. Accordingly, I indicate in each result whether I use the distribution of families in the subsidized equilibrium or in the baseline equilibrium.

2.4.1 Welfare Gains: Overall and Decomposition

I tabulate gains in welfare ex ante (for the unborn), and ex post (by age, skill, marital status, and gender groups). When describing and analyzing the distribution of welfare gains in the economy under different subsidies, I have separated adults into two groups: low and high skill. Low skill is defined as those below the 20th percentile of hourly wages in the baseline equilibrium. This procedure allows for more heterogeneity in the high-skilled group than the low-skilled group, but a finer decomposition does not reveal a qualitatively different result from the one I discuss below.

Ex Ante Welfare Gains

Table 2.6 reports ex ante welfare gains for children, overall and by initial skill group (low, medium, and high). In addition, the second column reports the subsidy level and the third column reports the marriage rate at that equilibrium. All welfare gains reported in the last four columns are in consumption-equivalent units, with expectations taken over the endogenous distribution of families in the subsidized equilibrium and the exogenous distribution of initial skill.

Overall, universal subsidies yield welfare gains that are 3.5 percentage points higher than subsidies to single mothers, and 3.9 percentage points higher than subsidies to poor families. The subsidy levels that achieve maximal gains under the different elibility rules are lowest for universal subsidies at 70%; for the two targeted policies I consider, they are 85%. Marriage rates rise under a universal subsidy and decrease under the two targeted subsidies; by making child care cheaper for any potential match, the subsidy lowers the threshold strategy of the marriage decision rule. By comparison, targeted child care subsidies raise the value of being a single parent by more. The reason it does so for the single mother subsidy is clear; for subsidies to the poor, it is harder to qualify for the subsidy as a couple than as a single mother (see Table 2.5 for a comparison of the composition of the poor in the model and in the ECLS-B).

Table 2.6: Ex Ante Welfare Gains, Subsidized Family Distribution

Eligibility	τ_n^*	Marriage rate	Welfare Gain Average	Welfare Gain by Skill Low	Medium	High
Universal	70%	100%	5.9	6.4	5.8	5.6
Single Mothers	85%	76%	2.4	3.6	2.2	1.5
Poor Families	85%	74%	2.0	3.5	1.8	1.0

Notes: Table 2.6 reports, by eligibility rule, the welfare-maximizing subsidy level, the marriage rate, and welfare gains on average and by initial skill of the child. Welfare gains are computed using expected gains over outcomes, using the distribution of families in the subsidized equilibrium, and are reported in percent gain of consumption-equivalent units that are rounded to the nearest tenth of one percent. Universal subsidies yield the highest gains, followed by single mother subsidies and subsidies to the poor. Gains are decreasing in initial skill.

In Table 2.7, I present welfare gains using the distribution of families at the baseline equilibrium. This is a relevant set of statistics because, conceivably, these are the families to whom policy makers are answerable when the policy is implemented. By comparing Table 2.6 with Table 2.7, it is evident that the ranking remains the same, with the level of gains almost identical. This is despite large change in economic aggregates and the distribution of families due the subsidies, as discussed in Tables 2.8 and 2.9.

Table 2.7: Ex Ante Welfare Gains, Baseline Family Distribution

		Welfare Gain	Welfare Gain by Skill		
Eligibility	τ_n^*	Average	Low	Medium	High
Universal	70%	5.9	6.4	5.9	5.5
Single Mothers	85%	2.1	3.3	1.9	1.4
Poor Families	85%	1.8	3.1	1.5	1.0

Notes: Table 2.7 reports, by eligibility rule, the welfare-maximizing subsidy level and welfare gains on average and by initial skill of the child. Welfare gains are computed as expected welfare gains over outcomes, using the distribution of families in the baseline equilibrium, and are reported in percent gain of consumption-equivalent units that are rounded to the nearest tenth of one percent. The ranking of subsidies is the same as in Table 2.6; also as in Table 2.6, gains are decreasing in initial skill.

Aggregate Moments: Comparison Across Eligibility Rules

Table 2.8 contains aggregate moments and equilibrium objects under the three eligibility rules at the welfare-maximizing level of τ_n. These aggregates are informative about the sources of welfare gains reported in Table 2.6 and Table 2.7. Under the single mother or universal subsidy, the labor income tax falls relative to the baseline by 1.8 and 0.2 percent, respectively. By comparison, a subsidy to the poor requires a 0.8 percent increase in labor income tax. Subsidies to the poor disincentivize skill investment, and so dampen the expansion in the labor income tax base compared to the other two eligibility rules (the percent change in H is lowest for subsidies to the poor). The price of non-parental care reflects changes in the average hourly wage under each eligibility rule: it rises the most under the universal subsidy (a 12 percent increase) and the least under the subsidy targeted to the poor (a 5.7 percent increase). This reflects the magnitude of the change in the average wage, which follows the same ranking as the change in p_n.

Table 2.8: Macroeconomic Moments and Equilibrium Objects Across Eligibility Rules

	r	τ_y	p_n	Y	C	H	K	L	N	Q_f	Q_m
Universal	0.999	0.998	1.120	1.249	1.240	1.245	1.257	0.994	1.893	1.132	1.029
Single Mothers	1.000	0.982	1.079	1.055	1.054	1.053	1.059	1.001	1.095	0.894	0.927
Poor Families	0.999	1.008	1.057	1.023	1.023	1.019	1.031	1.003	1.108	0.855	0.873

Notes: Table 2.8 reports macroeconomic moments and equilibrium objects across eligibility rules at the welfare-maximizing level of τ_n. Moments are reported as fractions of the baseline level, to facilitate interpretation in units of percentage change, and are rounded to the third decimal place (nearest tenth of a percent change). Recall, from the Model section, that the price of non-parental care is not an aggregate endogenous state but simply a function of it in this model. Moving from left to right, the columns report the interest rate, the price of child care (pre-subsidy), aggregate output, consumption, labor efficiency units supplied, capital stock, leisure, non-parental care time, quality time from fathers, and quality time from mothers. All aggregates use the distribution over families in the subsidized equilibrium.

Output, consumption, hours, the capital stock, child care use, and parental time investments (columns 5 to 8 and 10 to 12 of Table 2.8) increase the most under the universal subsidy. By contrast, leisure decreases under the universal subsidy, and average parental time investment levels decrease under the targeted subsidies. The increase in the labor income tax rate under poverty-tested subsidies is necessary because the expansion in the supply of labor efficiency units is low, while the increase in demand for child care is high enough that the costs of running the child care subsidy program offset the gains resulting from expansions in the labor income tax base.

Note that the estimated CES complementarity between parental time and non-parental time in the production of child skill means that, for a fixed level of parenting productivity, a change in the ratio of prices does not get completely reflected as a change in the ratio of inputs: that is, parenting time will not be completely crowded out by non-parental care as the subsidy increases. However, as the distribution of skill in the population increases, the opportunity cost of parenting time also goes up, while the productivity of non-parental time remains fixed. The ratio of time inputs into child skill development will reflect the increased value of parental time, as well as the lowered price of non-parental child care. Note that inputs from parents only increase under a universal subsidy. Parental time investments increase under the universal subsidy, but not under the targeted subsidies, because the marriage rate increases in the former and not the latter. Married couples in turn use a technology that requires higher levels of parental time inputs. Inputs from fathers fall in the two targeted subsidies due to decreases in the marriage rate, while inputs from mothers fall both because of lower marriage rates and because the composition of inputs adjusts to the subsidy on child care. Although the latter force still applies under universal subsidies, the higher marriage rate that results under that eligibility rule means that more mothers are parenting in a couple, and married mothers contribute more hours than single mothers do (see Table 3.1 in the Estimation section).

Table 2.9: Income and Earnings Levels and Gini Coefficient Across Eligibility Rules

	Income		Earnings		Pre-tax Income		Pre-tax Earnings	
	mean	Gini	mean	Gini	mean	Gini	mean	Gini
Universal	1.235	0.493	1.248	0.823	1.236	0.436	1.247	0.823
Single Mothers	1.054	1.123	1.056	1.040	1.052	1.142	1.054	1.040
Poor Families	1.023	1.186	1.020	1.063	1.023	1.212	1.021	1.063

Notes: Table 2.9 reports average income and earnings, both before and after tax, as well as the Gini coefficient for each measure. Moments are reported as fractions of the baseline level, to facilitate interpretation in units of percentage change, and are rounded to the third decimal place (nearest tenth of a percent change). Before normalizing, averages are calculated using the distribution over families in the subsidized equilibrium. Income and earnings are reported after labor income taxes are applied. Income is the sum of labor earnings, income from wealth, and lump-sum transfers, net of labor income taxes. Pre-tax income contains lump-sum transfers. Relative to the baseline, both income and earnings increase the most under the universal subsidy. Inequality as measured by the Gini coefficient decreases under the universal subsidy and increases under the other two eligibility rules.

Table 2.9 describes the behavior of the earnings and family income distributions relative to the baseline under the three eligibility rules. Universal subsidies increase average earnings and income the most, and they do so while decreasing inequality. For the two targeted subsidies, however, inequality increases even as levels of earnings and income increase. Targeted subsidies increase inequality by increasing the number of single parents, who have lower family incomes, and by distorting the skill investment decision at the eligiblity threshold in the case of subsidies to the poor. Note that after-tax gains in earnings and income under subsidies targeted to single mothers are higher than pretax levels, while for subsidies to the poor the opposite is true, reflecting different equilibrium changes in labor income taxes under the two eligibility rules. Note also that, by comparing the Gini coefficients of before- and after-tax income, one can see that the labor income tax acts to decrease dispersion. The dispersion of earnings is unaffected because the labor income tax is linear.

Decomposition of Welfare Gains

Table 2.10 decomposes welfare gains by marital status and age across eligibility rules at the welfare-maximizing level of τ_n. Besides benefiting directly from receiving the subsidy, gains during parenthood occur due to altruistic enjoyment of insurance to marriage market risk, even for ineligible parents, and because of any decrease in the labor income tax resulting from the subsidy. Later in life, families that were able to save more during parenthood because of lower parenting costs can afford to finance more consumption and so see higher gains. Single fathers experience much lower gains from this particular source, and this is reflected in concentration of welfare gains for that group during the parenting phase.

Table 2.10: Welfare Gains by Age and Marital Status

Universal Subsidy

	Average	Married Couples	Single Fathers	Single Mothers
Parenthood	5.7	5.7	3.9	7.6
Worker	1.6	1.6	0.0	1.7
Old	1.4	1.4	-0.1	1.8

Targeted: Single Mothers

	Average	Married Couples	Single Fathers	Single Mothers
Parenthood	2.2	1.2	1.9	8.4
Worker	0.6	0.4	0.1	2.5
Old	0.5	0.2	0.0	2.6

Targeted: Poor Families

	Average	Married Couples	Single Fathers	Single Mothers
Parenthood	2.0	1.2	1.5	6.8
Worker	0.3	0.3	-0.1	0.4
Old	0.2	0.3	-0.1	0.3

Notes: Table 2.10 shows the welfare gains for each eligibility rule at the optimal subsidy level for that rule. Gains are reported at each age, overall and by family structures. Welfare gains are in percentage point gains of consumption equivalent units, and are aggregated using the distribution of families at the subsidized equilibrium. Gains are rounded to the first decimal point (nearest tenth of a percent gain). Under all three eligibility rules, single mothers gain the most at all phases of life compared to single fathers and married couples. Because subsidies to the poor require a slight increase in the labor income tax rate and result in the lowest increase in earnings (see Table 2.9), gains are lowest for this eligibility rule.

Single mothers benefit more than married couples under all three eligibility rules I consider. This is because a given level of subsidy has a larger effect on their price of investment compared with couples (due to the point estimates of the skill accumulation technology, which I present and discuss in the Estimation section). Compounding this, under either of the targeted subsidies single mothers are far more likely to receive the subsidies than couples. Couples under the two targeted subsidy regimes benefit altruistically from marriage market insurance to their children, and directly from any equilibrium decrease in the labor income tax.

Table 2.11: Welfare Gains by Marital Status, Gender, Age and Skill

Universal Subsidy	Married Couples		Single Fathers		Single Mothers	
	Low	High	Low	High	Low	High
Parenthood	6.0	5.7	3.9	3.9	7.8	7.5
Worker	1.5	1.6	0.0	0.0	1.5	1.7
Old	1.4	1.4	-0.1	-0.1	1.6	1.9

Targeted: Single Mothers	Married Couples		Single Fathers		Single Mothers	
	Low	High	Low	High	Low	High
Parenthood	1.3	1.2	1.9	1.9	8.5	8.1
Worker	0.5	0.4	0.1	0.1	2.1	3.8
Old	0.4	0.2	0.0	0.0	2.3	3.9

Targeted: Poor Families	Married Couples		Single Fathers		Single Mothers	
	Low	High	Low	High	Low	High
Parenthood	2.0	0.8	1.6	1.6	8.2	0.8
Worker	0.6	0.2	-0.1	-0.1	0.3	0.3
Old	0.6	0.1	-0.1	-0.1	0.3	0.3

Notes: Table 2.11 shows the welfare gains for each eligibility rule at the optimal subsidy level for that rule. Gains are reported at each age, overall and by family structures. Welfare gains are in percentage point gains of consumption equivalent units, and are aggregated using the distribution of families at the subsidized equilibrium. Gains are rounded to the first decimal point (nearest tenth of a percent gain). Across eligibility rules, all parents benefit, even the ineligible: this is due to decreases in the labor income tax under universal and single-mother subsidies, as well as the marriage market insurance the subsidy provides under all three eligibility rules (which parents internalize due to altruism). The gains in the single father column appear identical because of rounding.

Table 2.11 provies the finest decomposition of welfare gains that I analyze. Here, low skill is the bottom 20 percent of the skill distribution in the economy. Under all three subsidies, all parents benefit, but single fathers of any skill level see welfare losses later in life. As with ex ante welfare gains, welfare gains for adults at any phase of life are decreasing in skill. Overall, subsidy levels that maximize ex ante welfare for a given eligibility rule are higher under targeted subsidies than under the universal subsidy. The optimal level of the subsidy is determined by balancing gains from insurance and increases in skill with the cost burden of funding the program. With targeted subsidies, the costs are lower and it takes higher levels of the subsidy for these costs to begin to offset gains from insurance and increases in skill. Insurance is an important source of welfare gains here because everyone in this economy faces risks over their own initial skill, the initial skill of their child, and the skill of their potential spouse. Without intervention, skill investment is the only way to insure against those risks. Welfare gains continue to increase with the subsidy until it is quite high (especially under the targeted eligibility rules) because it provides insurance against poor outcomes, which in this environment can happen to anyone.

2.4.2 Model Specification: Effects on Welfare Gains

Welfare gains from the subsidy schemes that I study are fairly large, on the order of 2% to 6% (Tables 2.6 and 2.7). These are comparable the existing literature on the welfare gains from subsidizing skill accumulation in early childhood, or even later in life (Daruich (2017), Abbott, Gallipolli, Meghir, and Violante (2018)). As discussed in the Model section, a child care subsidy in this environment yields welfare gains from four main sources: it insures against a low initial skill for the child, against being born into a family with fewer resources, and against a poor outcome in a frictional marriage market. In addition, it partially addresses a fiscal externality to parental investments in their child's skill.[7]

Because I have found that my data do not allow me to predict the initial skill of the child using family attributes, in the model I have not made the distribution from which the initial child skill is drawn depend on parental attributes. This choice magnifies the degree of risk a child faces once they know their initial skill but are still uncertain about their family: if skill were very persistent across generations, knowing your initial skill would be very informative about the sort of parents you would have. Likewise, if initial skill were drawn from a distribution that was endogenous, gains from insuring this risk would be realized by directly affecting that distribution. Consequently, the welfare gains from universal subsidies to children before they know their initial skill or family are quite large in magnitude: these are unavoidable and important shocks to lifetime utility.

In addition, because I have found that I cannot predict initial skill with the measures of

[7] The term "fiscal externality" is viewed as a misnomer by some economists. Here I use it in the sense that a child care subsidy affects those not receiving the subsidy (those not parenting, or parents who are not eligible) indirectly by allowing for a decrease in the labor income tax in equilibrium. See Buchanan (1966) and Browning (1999).

prenatal investment that I have in the ECLS-B, I have not modelled choices of parents that can affect the initial skill of the child (see the appendix for regressions motivating this and other modelling choices).[8] In addition to the fact that initial skill is not persistent across generations, this means that all parents face the same risk over the initial skill of their children: they differ in their ability to compensate for a low initial skill by investing more. A model with persistence in initial skill would reduce this source of risk and open up other policy discussions (see Abbott, Gallipolli, Meghir, and Violante (2018), and Rustichini, Iacono, and McGue (2016)).

Risk from the marriage market arises from several sources. The marriage market that I model is a simple one: potential spouses are matched randomly, and positive assortative matching by skill occurs because adults have the power to reject the spouse they match with. A marriage market where search was more directed would reduce the risk faced by a new adult over the potential spouse they meet, by giving them some control over it. Here, I only allow parents to provide insurance against marriage market risk by ensuring that their child has enough skill to not be rejected by the spouse they meet. The individual cannot affect her chances, but must appear on the marriage market with the skill she has as an adult and make her decision. I interpret the risk added by the presence of the marriage market as an upper bound: with a directed search framework, this risk would be mitigated. Another source of risk in the marriage market stems from the way I set up the married couple problem. Spouses are not allowed to bargain over the share of household utility they receive. Instead I emphasize economies of scale in consumption (through consumption equivalence scales), and gains from teamwork in investment in the couple's children through the skill technology. By contrast, a framework with bargaining would have allowed the spouse who gains the most from the match relative to being single to convince their potential spouse to marry them, with the promise of a larger fraction of household utility in return (Choo and Siow (2006), Reynoso (2017)). These endogenous shares would act like prices. The missing market for transferring utility between spouses is absorbed into the risk faced on the marriage market, and increases the gains from insuring against a poor outcome there.

Finally, in the aggregate a larger labor income tax base allows the labor income tax rate to be lowered in equilibrium to balance the government budget constraint. The means that expansions in the labor income tax base due to the child care subsidy can increase welfare for everyone in society by allowing the labor income tax to decrease.

[8] The assumption of no prenatal investments and the assumption of no genetic persistence in skill are distinct. For an excellent discussion of studies on the role of prenatal care, see Corman, Dave, and Reichman (2018). For a discussion of studies on the relationship between genes and cognitive skills, see National Scientific Council on the Developing Child (2010) and the citations therein.

2.5 Conclusion

In this paper, I build and estimate a framework that incorporates a relevant dimension along which to target child care subsidies in general equilibrium: family structure. Specifically, I allow prices, tax rates, the distribution of skill, and family formation to adjust endogenously in response to the child care subsidy. After estimating the technologies that single mothers and couples use to invest in their children, I apply the model to compare the welfare effects of universal subsidies to child care with subsidies targeted at the poor and to one-parent families with young children. I find that universal child care subsidies yield ex ante welfare gains of 5.9 percentage points in consumption equivalent units. These gains are 3.5 and 3.9 percentage points higher than subsidies to single mothers or the the poor, respectively. Compared to the two targeted subsidies I consider, universal subsidies are a better policy because they more fully insure newborns against the risks they face without disincentivizing skill investment.

This analysis could be extended to incorporate the effects of different eligibility rules over the transition from one steady-state to another. The analysis I conduct here is a comparison of steady-states: in the transition, gains from expansions in the population's distribution of skill will take time to realize, and generations who incur the costs of funding the subsidy without themselves enjoying the gains in skill it affords will see welfare losses. A steady-state comparison does not account for these dynamics. In addition, the non-parental care sector can be further elaborated, so that the elasticity of supply for non-parental care can be disciplined and its implications for large-scale subsidies examined (Blau (1993)). Finally, more structure could be imposed on the single father problem specification. Most of the work done on this front has focused on divorced spouses, rather than on those who never marry, as in my model (Del Boca and Flinn (1995), Tartari (2015)).

As it stands, this paper provides a useful framework for analyzing child care policies in an environment with heterogeneous family structures, with implications for how such policies will affect the marriage rate, tax levels, and welfare. This paper also provides new estimates on how parental time inputs interact with non-parental child care time to affect child skill accumulation during early childhood, for one- and two-parent families.

Chapter 3

Estimation of the Skill Accumulation Technologies

3.1 Introduction

In this chapter, I describe in detail how the skill accumulation technologies used in the model of Chapter 2 are estimated. First, I give the functional forms of the technologies used by one- and two-parent families. Then, using the first order conditions of the family problems from the model of Chapter 2, I derive estimation equations and map their dependent and independent variables to empirical analogs. I motivate the use of a fixed effects estimator my demonstrating that unobserved parenting productivies satisfy the requirements of that estimator: their are linearally seperable in the estimation equation. Next, I describe the ECLS-B and the American Time Use Survey (ATUS), which I use to construct the necessary variables for the estimation equations. This portion of the chapter includes summary statistics of the data and a discussion of the empirical compositional differences between one- and two-parent families. Finally, I report the results for the estimation of skill accumulation technologies, discuss their implications for the results in Chapter 2, and compare them with other findings in the literature.

3.2 Functional Forms of the Skill Accumulation Technologies

I specify the skill accumulation technologies to be nested constant elasticity of substitution (CES) functions with inputs of non-parental time, mother time, and father time (which aggregate to investment I_{type}) and between investment and the current stock of skill (which aggregate to tomorrow's skill). Specifically, the functional forms of the dynamic equation for skill accumulation, and how investment is generated from time inputs for the two family structures, are as follows:

$$\begin{aligned} f_{type}(\cdot) &= \left[\upsilon\left(\lambda I_{type}\right)^{\xi} + (1-\upsilon)\,\theta_c^{\xi}\right]^{\frac{1}{\xi}}, \quad type \in \{s, mc\} && (3.1) \\ I_s &= \left[\gamma_s\left(\theta_m q_m\right)^{\eta_s} + (1-\gamma_s)\,(n)^{\eta_s}\right]^{\frac{1}{\eta_s}} && (3.2) \\ I_{mc} &= \left[(1-\alpha)\left(\gamma\left(\theta_m q_m\right)^{\eta} + (1-\gamma)\,(n)^{\eta}\right)^{\frac{\rho}{\eta}} + \alpha\left(\theta_f q_f\right)^{\rho}\right]^{\frac{1}{\rho}} && (3.3) \end{aligned}$$

The parameters γ, γ_s and α control the relative level of inputs given a price ratio. The values of η, η_s, and ρ control the percentage change in the ratio of inputs for a percentage change in the ratio of their prices. With this parameterization, I allow for mother time and non-parental child care time to interact differently across household structures (i.e., I do not impose $\gamma = \gamma_s$ and $\eta = \eta_s$). [1]

3.3 Estimation of Skill Accumulation Technologies

In this section, I explain how I estimate the parameters of the skill technology from the ECLS-B dataset. The panel nature of the data allows me to use a fixed-effects estimator to control for heterogeneous productivity shifters on parental time inputs. The equations that I use to estimate the skill accumulation technology parameters are derived from the first order conditions of the parenting problem with respect to the quality time of the father and mother, and non-parental child care time. I assume that the aggregator of maternal time and non-parental time is the same functional form for both single mothers and married couples. Married couples differ from single mothers because of the time contribution of the father, which is modelled as an outer aggregator of father time and maternal/non-parental time. After taking first order conditions of the parenting problem with respect to the time investment choices, I take ratios of those equations and then logs. The result is a system of four linear equations. After deriving these equations with model notation, I explain how the objects in my model map into variables in the ECLS-B and motivate my use of the fixed effects estimator. I then provide the estimation equations in terms of observed variables in the ECLS-B. My derivation of the estimation equations is similar to the method of Lee and Seshadri (2018).

[1] A different and common way of specifying the investment aggregator is to include money (goods) and parental time inputs. See the appendix for an exercise where I measure the contribution of child care costs to expenditures on children in different age groups, using the 2001 PSID and the 2002 PSID CDS. I find that child care costs are a sizeable compoenent of money spent on children by any measure of spending on children I consider (the share ranges from 50 to 70 percent of total spending). My specification makes explicit how expenditures on children affect child skill accumulation through child time use.

3.3.1 Estimation Equations

The equations for the investment aggregators of singles and couples are given by (3.3) and (3.3). The partial derivatives of these equations enter the first order conditions of the parenting problem with respect to q and n for single mothers and q_m, q_f and n for married couples. After taking ratios and then logs of these first order conditions, I get the following linear equations, one for single mothers and two for married couples. It is apparent that the parenting productivities can be separated linearly from the terms of interest.

$$\ln\left(\frac{q}{n}\right) = \left(\frac{1}{\eta_s - 1}\right)\ln\left[\frac{w\theta}{(1-\tau_n)\,p_n}\right] + \left(\frac{1}{\eta_s - 1}\right)\ln\left[\frac{1-\gamma_s}{\gamma_s}\right] - \left(\frac{\eta_s}{\eta_s - 1}\right)\ln(\theta) \tag{3.4}$$

$$\ln\left(\frac{q_m}{n}\right) = \left(\frac{1}{\eta - 1}\right)\ln\left[\frac{w\theta_m}{(1-\tau_n)\,p_n}\right] + \left(\frac{1}{\eta - 1}\right)\ln\left[\frac{1-\gamma}{\gamma}\right] - \left(\frac{\eta}{\eta - 1}\right)\ln(\theta_m) \tag{3.5}$$

$$\begin{aligned}\ln\left(\frac{q_f}{n}\right) &= \left(\frac{1}{\rho - 1}\right)\ln\left[\frac{w\theta_f}{(1-\tau_n)\,p_n}\right] + \left(\frac{\eta-\rho}{\eta(\rho - 1)}\right)\ln\left(\frac{q_m}{n}\frac{w\theta_m}{(1-\tau_n)\,p_n} + 1\right) \\ &+ \left(\frac{1}{\rho - 1}\right)\ln\left[\frac{(1-\alpha)}{\alpha(1-\gamma)^{\frac{\rho}{\eta}}}\right] - \left(\frac{\rho}{\rho - 1}\right)\ln(\theta_f)\end{aligned} \tag{3.6}$$

In the first equation q and n are the quality time and non-parental child care time chosen by single mothers, respectively, $w\theta$ is the hourly wage of single mothers, p_n is the price of non-parental child care, and θ is the parenting productivity of single mothers. In the second and third equations, q_m, q_f, and n are the quality time inputs of mothers and fathers and the non-parental child care time chosen by married couples, $w\theta_m$ is the hourly wage of the mother, $w\theta_f$ is the hourly wage of the father, p_n is the price of non-parental child care time, and $\theta, \theta_m, \theta_f$ are the parenting productivites of single mothers, married mothers, and married fathers, respectively.

To derive estimation equations for the skill aggregator equation (3.2), I impose intertemporal cost minimization, similar to the approach of Lee and Seshadri (2018). This means that I assume parents set the ratio of the marginal costs of investment (the prices of investment in each period, as derived above) equal to the ratio of marginal productivities of investment in each period, appropriately discounting the costs using interest rates. The result of the following steps will be an equation that expresses the ratio of expenditures over two consecutive periods on the left-hand side, and the ratio of investment prices over two consecutive periods on the right-hand side. First, I explain how I derive the price of skill investment for single and married couples. Second, I construct the equation I will use to estimate parameters υ and ξ in equation (3.2).

The total expenditures on investment, $X_{type,j}$, is the sum of foregone earnings and non-parental child care costs for the time invested in the child. For each family type, the expression for this cost is:

$$X_s = (1-\tau_n) p_n n + w\theta q_m \tag{3.7}$$

$$X_{mc} = (1-\tau_n) p_n n + w\theta_f q_f + w\theta_m q_m \tag{3.8}$$

Investment expenditures purchase investment input into skill accumulation at a price Λ_{type}:

$$X_{type} = \Lambda_{type} I_{type} \tag{3.9}$$

An analytical expression for the price of investment Λ_{type} can be derived by taking the ratio of X_{type} and I_{type}. The solution will be a function of parental skill and the prices of inputs, which are values the parent takes as given in this model.

For couples, the price of investment Λ_{mc} is:

$$\Lambda_{mc} = \frac{(1-\tau_n) p_n + w\theta_f \frac{1}{\theta_f} \Psi_{2,mc} + w\theta_m \frac{1}{\theta_m} \Psi_{1,mc}}{\Psi_{4,mc}} \tag{3.10}$$

where

$$\begin{aligned}
\Psi_{1,mc} &\equiv \left[\frac{w\theta_m}{(1-\tau_n) p_n} \frac{(1-\gamma)}{\gamma} \frac{1}{\theta_m}\right]^{\frac{1}{\eta-1}} \\
\Psi_{2,mc} &\equiv \left[\frac{1}{\theta_f} \frac{(1-\alpha)(1-\gamma)}{\alpha} \frac{w\theta_f}{(1-\tau_n) p_n} (\Psi_{3,mc})^{\rho-\eta}\right]^{\frac{1}{\rho-1}} \\
\Psi_{3,mc} &\equiv \left(\gamma (\Psi_{1,mc})^{\eta} + 1 - \gamma\right)^{\frac{1}{\eta}} \\
\Psi_{4,mc} &\equiv \left[\alpha (\Psi_{2,mc})^{\rho} + (1-\alpha)(\Psi_{3,mc})^{\rho}\right]^{\frac{1}{\rho}}
\end{aligned}$$

For single mothers, the price of investment Λ_s is:

$$\Lambda_s = \frac{(1-\tau_n) p_n + w\theta \frac{1}{\theta} \Psi_{1,s}}{\Psi_{2,s}} \tag{3.11}$$

where

$$\begin{aligned}
\Psi_{1,s} &= \left[\frac{1}{\theta} \frac{(1-\gamma_s) w\theta}{\gamma_s (1-\tau_n) p_n}\right]^{\frac{1}{\eta_s-1}} \\
\Psi_{2,s} &\equiv \left(\gamma_s (\Psi_{1,s})^{\eta_s} + 1 - \gamma_s\right)^{\frac{1}{\eta_s}}
\end{aligned}$$

The assumption of intertemporal cost minimization implies that for two consecutive observations in the data $t-1$ and t, the ratio of the marginal prices of investment equals the ratio of marginal productivities of that investment. Notice that here I am allowing childhood to have multiple periods as in the data, while in my model I assume it is a single period. This is why I have shifted from using j to index age in the model to using t to reference the period in the data. In this sense, my estimates of the skill accumulation technology parameters are quasi-structural. Now, for the objects θ_c, X_{type}, I_{type}, and Λ_{type}, I add a time subscript t or $t-1$.

$$\frac{\left(\frac{1}{1+r_t}\right)\Lambda_{type,t}}{\left(\frac{1}{1+r_{t-1}}\right)\Lambda_{type,t-1}} = \frac{\frac{\partial\theta_{c,t+1}}{\partial I_{type,t}}}{\frac{\partial\theta_{j+1}}{\partial I_{type,j-1}}} \tag{3.12}$$

where $type \in \{s, mc\}$. Substituting $\frac{X_{type,t}}{\Lambda_{type,t}}$ for $I_{type,t}$ and taking logs of both sides of the above equation gives a linear equation with the ratio of investment expenses on the left-hand side and the ratio of prices on the right-hand side:

$$\ln\left(\frac{X_{type,t}}{X_{type,t-1}}\right) = \frac{1}{\xi-1}\ln\left[1-\upsilon\right] + \frac{\xi}{\xi-1}\ln\left(\frac{\Lambda_{type,t}}{\Lambda_{type,t-1}}\right) + \frac{1}{\xi-1}\ln\left[\frac{1+r_t}{1+r_{t-1}}\right] \tag{3.13}$$

To control for the last term, which is a ratio of interest rates across periods, I assume a yearly interest rate of 4% (consistent with the risk-free rate that gives the discount factor β) and adjust the values $1+r_t$ in each period t to be proportional to its distance from 0, the period when the child is born. This means that at age 9 months, the first wave of the survey, the interest rate is $\frac{3}{4}\times(1.04)$, while in the second wave of the survey (when the child is 2 years old) this value is $2\times(1.04)$. Notice that this last equation does not contain an additive constant that is cancelled out by a fixed effects estimator. Note also that I did not cancel parenting and labor market productivities in the definitions of Λ_{type}, in order to make the mapping from the model to the data more straightforward in the next section. When constructing prices, I recover the parenting productivity from the fixed effects estimations used to find the values of η, γ, η_s, and γ_s. I use those family-level values to build the price of investment at the family level in each period t.

Mapping Model Objects to Objects in the Data In the data section below, I explain in detail the procedure I use to prepare the ECLS-B data for estimation. After applying the procedure I describe there, I observe the following attributes of family i in wave t: hourly wages $\tilde{w}_{i,f,t}, \tilde{w}_{i,m,t}$ for father f or mother m, hourly price of child care $\tilde{p}_{i,t}$ (which differs across families and across waves), quantity of child care purchased $\tilde{n}_{i,t}$ in units of hours per week, and hours of quality time contributed by the father f and the mother m: $\tilde{q}_{i,f,t}, \tilde{q}_{i,m,t}$. Here I also put the m subscripts on the single mother variables in the data, which is a departure from the model notation.

For couples, I estimate η and γ first, and then predict the ratio of mother time and non-parental child care time at the family level (incorporating the fixed effect levels). This is then substituted into the right-hand side of the estimation equation for ρ, α with father time and non-parental child care time on the left-hand side. For the last estimation equation, I construct investment prices $\tilde{\Lambda}_{type,i,t}$ and expenditures $\tilde{X}_{type,i,t}$ for each family in each wave by using the analogous variables from the data in equations (3.8) and (3.8). Individual-level parenting productivities θ, θ_m, and θ_f are not observed in the ECLS-B. I assume that these are time-invariant but can vary across individuals. I denote these unobserved parenting productivities with $\tilde{\theta}_{i,m}$ for the mother and $\tilde{\theta}_{i,f}$ for the father. Rewriting the four equations derived in the previous sections using notation for variables in the ECLS-B yields:

$$\ln\left(\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}\right) = \left(\frac{1}{\eta_s-1}\right)\ln\left[\frac{\tilde{w}_{i,m,t}}{\tilde{p}_{i,t}}\right]+\left(\frac{1}{\eta_s-1}\right)\ln\left[\frac{1-\gamma_s}{\gamma_s}\right]-\left(\frac{\eta_s}{\eta_s-1}\right)\ln\left(\tilde{\theta}_{i,m}\right) \tag{3.14}$$

$$\ln\left(\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}\right) = \left(\frac{1}{\eta-1}\right)\ln\left[\frac{\tilde{p}_{i,t}}{\tilde{w}_{i,m,t}}\right]+\left(\frac{1}{\eta-1}\right)\ln\left[\frac{1-\gamma}{\gamma}\right]-\left(\frac{\eta}{\eta-1}\right)\ln\left(\tilde{\theta}_{i,m}\right) \tag{3.15}$$

$$\begin{aligned}\ln\left(\frac{\tilde{q}_{i,f,t}}{\tilde{n}_{i,t}}\right) &= \left(\frac{1}{\rho-1}\right)\ln\left[\frac{\tilde{p}_{i,t}}{\tilde{w}_{i,f,t}}\right]+\left(\frac{\eta-\rho}{\eta(\rho-1)}\right)\ln\left(\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}\frac{\tilde{w}_{i,m,t}}{\tilde{p}_{i,t}}+1\right) \\ &+ \left(\frac{1}{\rho-1}\right)\ln\left[\frac{(1-\alpha)}{\alpha(1-\gamma)^{\frac{\rho}{\eta}}}\right]-\left(\frac{\rho}{\rho-1}\right)\ln\left(\tilde{\theta}_{i,f}\right)\end{aligned} \tag{3.16}$$

$$\ln\left(\frac{\tilde{X}_{type,i,t}}{\tilde{X}_{type,i,t-1}}\right) = \frac{1}{\xi-1}\ln[1-\upsilon]+\frac{\xi}{\xi-1}\ln\left(\frac{\tilde{\Lambda}_{type,i,t}}{\tilde{\Lambda}_{type,i,t-1}}\right)-\frac{1}{\xi-1}\ln\left[\frac{1+r_t}{1+r_{t-1}}\right] \tag{3.17}$$

Recovering the Parameters of Interest For estimation purposes, equations (3.14) to (3.17) are rewritten as:

$$\ln\left(\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}\right) = \beta_{1,0}+\beta_{1,1}\ln\left[\frac{\tilde{w}_{i,m,t}}{\tilde{p}_{i,t}}\right]+\beta_{1,2}\ln[\theta_{i,m}]+\epsilon_{i,t} \tag{3.18}$$

$$\ln\left(\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}\right) = \beta_{2,0}+\beta_{2,1}\ln\left[\frac{\tilde{w}_{i,m,t}}{\tilde{p}_{i,t}}\right]+\beta_{2,2}\ln[\theta_{i,m}]+\epsilon_{i,t} \tag{3.19}$$

$$\ln\left(\frac{\tilde{q}_{i,f,t}}{\tilde{n}_{i,t}}\right) = \beta_{3,0}+\beta_{3,1}\ln\left[\frac{\tilde{w}_{i,f,t}}{\tilde{p}_{i,t}}\right]+\beta_{3,2}\ln\left(\widehat{\frac{\tilde{q}_{i,m,t}}{\tilde{n}_{i,t}}}\frac{\tilde{w}_{i,m,t}}{\tilde{p}_{i,t}}+1\right)+\beta_{3,3}\ln[\theta_{i,f}]+\epsilon_{i,t} \tag{3.20}$$

$$\ln\left(\frac{\tilde{X}_{type,i,t}}{\tilde{X}_{type,i,t-1}}\right) = \beta_{4,0}+\beta_{4,1}\ln\left(\frac{\tilde{\Lambda}_{type,i,t}}{\tilde{\Lambda}_{type,i,t-1}}\right)+\beta_{4,2}\ln\left[\frac{1+r_t}{1+r_{t-1}}\right]+\epsilon_{i,t} \tag{3.21}$$

In an Ordinary Least Squares (OLS) estimation, the assumption that the residual was uncorrelated with the regressors would be violated in equations (3.18) to (3.20) if parenting productivities and labor market productivity were correlated (unobserved parenting productivities would be absorbed into the residual). Consequently, in regressions (3.18) to (3.20)) I use a fixed effects estimator, which controls for the time-invariant parenting productivities by subtracting out the family-level mean and adding back in the population mean. Values for the population mean are reported under an assumption about the population average of $\ln(\theta_i)$. Since the productivity of non-parental child care is normalized to 1, this level assumption is equivalent to one about the distribution of the log of relative parent/non-parent productivities in the population. That is, I assume the population average of the logged relative parenting productivities is 0. Given that I do not offer empirical discipline for this choice, I conduct several robustness checks in the appendix on the share parameters in the skill accumulation technologies.

The mapping from regression coefficients to the model's parameters is as follows:

$$\gamma_s = \frac{1}{\exp\left(\frac{\beta_{1,0}}{\beta_{1,1}}\right)+1} \quad , \quad \eta_s = \frac{1}{\beta_{1,1}} + 1$$

$$\gamma = \frac{1}{\exp\left(\frac{\beta_{2,0}}{\beta_{2,1}}\right)+1} \quad , \quad \eta = \frac{1}{\beta_{2,1}} + 1$$

$$\alpha = \frac{1}{\frac{\exp\left(\frac{\beta_{3,0}}{\beta_{3,1}}\right)}{1-\gamma}+1} \quad , \quad \rho = \frac{1}{\beta_{3,1}} + 1$$

$$\upsilon = 1 - \exp\left(\frac{\beta_{4,0}}{\beta_{4,1}-1}\right) \quad , \quad \xi = \frac{\beta_{4,1}}{\beta_{4,1}-1}$$

3.3.2 Data

The variables that I need to observe are the ones needed to construct the dependent and independent variables in the regression equations (3.18) to (3.19). The subscripts denote family i in wave t, with father f and mother m.

- Parental educational time inputs $(\tilde{q}_{i,f,t}, \tilde{q}_{i,m,t})$
- Non-parental child care time inputs $(\tilde{n}_{i,t})$
- Hourly wages $(\tilde{w}_{i,m,t}, \tilde{w}_{i,f,t})$
- Hourly non-parental child care prices $(\tilde{p}_{i,t})$

There are two data sources that I use to measure these variables: the ECLS-B, which is a panel dataset, and the American Time Use Survey (ATUS), which is a repeated cross-section sampled from the Current Population Survey (CPS).

The Early Childhood Longitudinal Study, Birth Cohort Data for hourly wages, hourly cost of child care, quality time from the parents in weekly frequency of activities, and non-parental child care time in hours per week come from the ECLS-B.

The ECLS-B reports incomes, the period of time over which the income was earned (a day, a week, two weeks, etc.), and the hours worked in a week. From these I can construct hourly wages for mothers and fathers, if they report these variables. If they did not report hours worked, I used part-time or full-time status to assign 30 or 40 hours worked per week, respectively. If they did not report earnings because they were out of the labor force (an issue confined mostly to mothers) I imputed their hourly wages using a regression of hourly wages on education, age, and age squared for the sample on which I could construct wages with reported income. I then evaluated this regression for observations where I could see education and age to get imputed hourly wages. Finally, to correct for taxes, I use the slopes (tax rates) from Table 2 of McGrattan and Prescott (2017). It is after-tax hourly income, not the directly reported pre-tax income, that I use in the estimation.

For the price of non-parental child care, I use weekly spending on each of the three main kinds of non-parental child care providers: relative, nonrelative, and center-based. Of these, I keep the primary source of non-parental child care. To calcuate cost per hour of non-parental child care, I use the total cost per time unit for the primary source of non-parental child care, and adjust it by the number of weeks that cost represents and the hours per week the child spends in that form of non-parental child care. After completing these steps, I have the price per hour of parental time and non-parental child care at the family level.

I define quality time as activities with the child that include talking and listening, singing, and reading to them. In the ECLS-B, these are reported as frequency per unit time. To convert these units into hours per week, I impute time per activity using the ATUS.

The American Time Use Survey Data on levels of time per activity for a parent with a given set of characteristics come from the ATUS. To impute levels of time per activity to the ECLS-B, I use a pooled sample from the ATUS from 2003 to 2016. This provides a time diary along with CPS variables on age, gender, marital status, labor force status, educational attainment, parental status, and child age. I restrict the sample to parents who are between 15 and 55, with a child 4 years or younger. I use information on gender, marital status (married/cohabiting or single), labor force status, and educational attainment, where educational attainment is discretized into those with less than a college degree or a college degree and higher. With this information, I group observations from the pooled sample by their characteristics along the aforementioned dimensions. For each group, I find survey-weighted averages of time spent on an activity for those who report engaging in it. The activities whose average duration I tabulate are time spent reading to the child and time spent talking with and listening to the child.

Imputation After linking parents in the ECLS-B with their appropriate group in the ATUS, I assign the level of time spent reading, and time spent talking and listening, to their respective activities in the ECLS-B; additionally, I assign the level of time spent talking and listening to the singing activity reported by parents in the ECLS-B. I do this because there is no singing activity reported in the ATUS. Once I have imputed time levels per activity associated with the ECLS-B sample, I aggregate levels of quality time per parent in each family in each wave. This gives me quality time investments, non-parental time investments, after-tax hourly wages, and non-parental child care prices at the child-family pair and wave level. I pool cohabiting and married couples in the data and refer to them collectively as "married couples." Single mothers are defined as mothers in the sample who are a primary caregiver and who do not have a significant other living in the household with them. In addition, I select only families with observations in all three of waves 1, 2, and 3. With this information, I can implement the estimation.

Estimating Sample Moments Moments from the estimation samples for couples and single mothers are presented in Table 3.1 and Table 3.2, respectively. I use wave 3 survey weights provided by the ECLS-B, which are designed to correct for attrition in the sample.

Table 3.1: Married and Cohabiting Couples in the ECLS-B

Waves 1-3		
Levels		
	mean	sd
Education Time Father: Hours per Week	5.55	2.94
Education Time Mother: Hours per Week	7.99	2.74
Non-parental Care: Hours per Week	30.47	13.94
Ratio of Time: Father/Mother	0.76	0.50
Ratio of Time: Mother/Child Care	0.41	0.50
Hourly Pay Mother, After Tax	13.90	11.82
Hourly Pay Father, After Tax	15.55	11.97
Hourly Price Child Care	3.93	4.22
Ratio: Hourly Price Child Care/AT Hourly Wage Mother	0.37	0.48
Age Resident Mother	33.05	5.60
Age Resident Father	34.94	6.09
Rates		
Mother: BA or higher	0.55	
Father: BA or higher	0.47	
Marriage Rate	0.92	
Below 100 % Poverty Line	0.01	
Below 185 % Poverty Line	0.08	
Observations	3100	

Note: Sample size is rounded to nearest 50, following National Center for Education Statistics (NCES) requirements.

Based on Table 3.1, the following qualitative points are apparent. First, fathers and mothers spend substantial time per week engaged in education time with their children. Within a couple, fathers invest on average 76% of the mother's time. This will be reflected in the complementarity estimates I find for father and mother time in my skill accumulation technology estimation. Next, I find that reported hours per week in non-parental child care are 30 on average, but there is a high variation in this level. The relative quantities of the two time inputs I measure in this sample, along with hourly prices for each, will translate into large estimated CES shares on non-parental child care. The ratio of mother time to non-parental child care is on average 0.41, but with a high variation. Married or cohabiting mothers and fathers have hourly wages that are about four times higher than the hourly price of child care they use, although there is large variation in all of these prices within my sample. On average, the price of child care is 37% of the mother's hourly wage. Poverty in the sample of married or cohabiting parents is not common: only 1% are below the poverty line, while only 8% are below 185% of that threshold. Although I pool cohabiting and married couples in my estimation sample, most of these families are married couples: the marriage rate is 92%. Both parents are on average in their 30s, and the fraction in this group with a college degree or more is about half for each gender.

Table 3.2: Single Mothers in the ECLS-B

Waves 1-3		
Levels		
	mean	sd
Education Time Mother: Hours per Week	7.08	2.97
Non-parental Care: Hours per Week	35.57	12.37
Ratio of Time: Mother/Child Care	0.26	0.28
Hourly Pay Mother, After Tax	9.13	10.78
Hourly Price Child Care	2.41	2.77
Ratio: Hourly Price Child Care/AT Hourly Wage Mother	0.32	0.40
Age Resident Mother	28.07	6.36
Rates		
Mother: BA or higher	0.14	
Below 100 % Poverty Line	0.22	
Below 185 % Poverty Line	0.42	
Observations	1350	

Note: Sample size is rounded to nearest 50, following NCES requirements.

Table reftab:SumstatsSMECLSB presents a similar set of statistics for single mothers. Single mothers spend only slightly less time than married mothers in educational activities with their children, but non-parental child care time is on average five hours higher than for couples. The ratio of mother's time to non-parental child care time is correspondingly lower than for couples, at 0.26 (compared to 0.41 for the latter). Within a family, the ratio of mother time to non-parental child care time is almost half that of married or cohabiting families. The poverty rate of single mothers is twenty times higher than couples, at 22%; for percent of single mothers below 185% of the poverty line is about five times higher, at 42%. On average, single mothers make about five dollars per hour less than mothers parenting in couples. The ratio of this price to the mothers hourly wage is five percent smaller than for married and cohabiting mothers. Finally, the age of single mothers is on average 5 years lower than married or cohabiting mothers, and their educational attainment is one third that of married mothers: only 14% of single mothers have a college degree or more.

The comparisons across these two estimation samples help to establish priors about what the estimates of the skill accumulation technologies of each should look like. Because single mothers are the only source of parental time for their child, and the amount of non-parental time purchased is so large, one expects to see a larger CES share for non-parental child care in the single mother problem, ceteris paribus (I will expand more on that later in this section).

From the estimation equations, one can see that the fixed effect correction adjusts for the logged parenting productivity θ_f or θ_m (the productivity of non-parental child care is normalized to 0). When I construct the predicted ratio of mother and non-parental child care time for couples in equation (3.20), I use fixed effects recovered from equations (3.19). When constructing investment for the estimation of the outermost aggregator in equation (3.21), I incorporate parental fixed effects recovered from (3.18) to (3.20).

For estimation weights, I used wave 3 weights—for the primary caregiver survey sample to estimate $\{\eta, \gamma\}$ and $\{\eta_s, \gamma_s\}$, and for the father survey sample to estimate ρ and α. Using a fixed effects estimator, a requirement for clustering standard errors is heterogeneity in treatment effects, which is not the case here (see Section 4 of Abadie, Athey, Imbens, and Wooldridge (2017)). However, because I impute hourly wages for parents who are not working using age and education, the wages of those observations may have errors that are correlated at the state level. This imputation is my motivation for clustering the standard errors at the level of state of residence.

3.3.3 Estimation Results

I estimate the mother/non-parental time aggregator separately for married and cohabiting mothers whose husbands also filled out a resident father questionnaire. In Table 3.4, this corresponds to models 1 and 2. Model 3 estimates the parameters governing how father and mother time combine, and model 4 reports estimates for the aggregator and investment and current skill. These are estimations of equations (3.18), (3.19), (3.20), and (3.21). Models 1, 2, and 3 use a fixed effects estimator, but model 4 uses an OLS estimator. This is because I cannot linearly separate the parenting productivities in that equation.

The nesting order I have chosen restricts the substitutability of father time with mother time or non-parental child care time to be the same. Alternative nestings would require that non-parental child care be equally substitutable with mother and father time, or that mother time be equally substitutable with father time and non-parental child care time. For couples, I chose the nesting that makes it most comparable with that of single mothers. I use this rule because there is not a clear ranking of alternative nestings by fit, and the parameter estimates for couples do not change in a statistically significant way by changing this restriction with one exception (see the appendix for estimates with alternative nestings).

Table 3.3: Estimation Results: Skill Accumulation Technology

	(1) Single Mother + Child Care	(2) Cohabiting Mother + Child Care	(3) Cohabiting Father + Not Father	(4) Outermost Estimation
$log(\frac{\text{after-tax hourly wage mother}}{\text{cost per hour child care}})$	-0.308***	-0.497***		
	(0.0856)	(0.0522)		
$log(\frac{\text{after-tax hourly wage father}}{\text{cost per hour child care}})$			-0.349***	
			(0.0509)	
log(composite term)			-0.599**	
			(0.199)	
$log(\frac{\Lambda_{type,t}}{\Lambda_{type,t-1}})$				0.545***
				(0.0298)
$log(\frac{r_{t-1}}{r_t})$				12.13***
				(0.674)
Constant	-1.117***	-0.583***	-0.836***	0.770***
	(0.131)	(0.0659)	(0.0923)	(0.0417)
R^2	.1356	.2039	.15	.5053
Observations	1350	3100	3100	2600

Notes: Standard errors are in parentheses, clustered at the level of state of residence.

Sample sizes rounded to nearest 50, following NCES requirements. Sample sizes are pooled across waves 1 to 3.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

These results yield the parameters in Table 3.4 for the skill accumulation technologies.

Table 3.4: Parameters of the Human Capital Accumulation Technology

ρ	α	η	γ	η_s	γ_s	ξ	υ
-1.86	0.07	-1.01	0.24	-2.25	0.03	-1.2	0.82
(0.42)	(0.03)	(0.21)	(0.05)	(0.90)	(0.04)	(0.14)	(0.03)

Note: Standard errors are in parentheses, clustered at the level of state of residence and calculated using the delta method.

Except for the single mother share γ_s, these parameter values are statistically different from zero at the 5% significance level. As for the single mother share, it has a p-value of 0.47. The fact that γ_s is small relative to γ is consistent with my intuition based on sample summary statics.

When I translate the point estimates of η, η_s, and ρ into elasticities of substitution, the estimation yields the values shown in Table 6.

Table 3.5: Elasticities of Substitution

Father + Mother $\frac{1}{1-\rho}$	Married Mothers + Non-parental $\frac{1}{1-\eta}$	Single Mothers + Non-parental $\frac{1}{1-\eta_s}$
0.35	0.50	0.31
(0.05)	(0.05)	(0.09)

Note: Standard errors are in parentheses, clustered at the level of state of residence.

If the ratio of father over mother wage increases by 1%, the ratio of father time to mother time invested in their child decreases by 0.35%. If the denominator of both ratios changes to non-parental child care, the reaction to a 1% change in the former ratio is the same. For married mothers, a 1% increase in the ratio of the mother's hourly wage to the price of non-parental child care decreases the ratio of her time to non-parental time invested in her child by 0.50%. She is more responsive than married men or single mothers to the price of non-parental child care. Single mothers are the least responsive: they would adjust the ratio of their time to non-parental child care time by only 0.31%. The share values, meanwhile, indicate that the level of her time would already be very low compared to the time inputs from couples raising children in this economy.

For intuition on the share parameters, consider a single mother whose hourly wage is exactly the same as the hourly price of non-parental child care care. Setting the prices equal to one another provides the clearest intuition for the role of the share parameter, and I chose to use the single mother technology because it has no outermost nesting like the married couple does around the ratio of prices of interest (which includes the price I subsidize in my analysis). The expression for the ratio of her quality time and non-parental child care time is:

$$\frac{q}{n} = \left[\frac{1-\gamma_s}{\gamma_s}\frac{w\theta}{(1-\tau_n)\,p_n}\right]^{\frac{1}{\eta_s-1}}$$

Imposing that the ratio of prices is 1 yields:

$$\frac{q}{n} = \left[\frac{1-\gamma_s}{\gamma_s}\right]^{\frac{1}{\eta_s-1}}$$

Solving for γ_s, we have

$$\gamma_s = \frac{1}{\left[\left(\frac{q}{n}\right)^{\eta_s-1}+1\right]}$$

Knowing the elasticity of substitution between her time and non-parental child care, the share parameter pins down the ratio of her time and the time in non-parental child care she chooses. If these two levels are equal when the prices are the same, then the share parameters will be $\frac{1}{2}$; if the single mother's quality time inputs exceed the time she purchases in non-parental child care, even when the prices are the same, then the weight on her time will be greater than $\frac{1}{2}$, and vice versa. The degree to which the share parameter needs to adjust in order to explain the ratio of input choices when the price of each input is the same depends on the value of the complementarity parameter η_s. The more complementary the two inputs are, the more the share parameter will need to adjust to explain a large difference in their input levels.

3.4 Discussion of Estimation Results

There are three main takeaways from the skill technology estimation. First, the share parameters on father time and single mother time inputs are very low. Even if their hourly wages were the same as the prices of the other inputs in their technology, ratios of investment inputs would still show a relatively low level of inputs coming from their quality time. This is independent of the composition of hourly wages in either group. By contrast, non-parental child care has a large share in the investment technologies of both one- and two-parent families. If this share were zero, changes in the price of non-parental child care incurred by families (due to a subsidy such as the ones I examine) would have no effect on the price of investment or the skill accumulation of children.

The second takeaway concerns the relative elasticities of the three parental inputs with respect to non-parental child care. A 1% increase in the ratio of a single mother's hourly wage to the price of non-parental child care causes her to adjust the ratio of her time input to non-parental child care by 0.31%. She does not heavily readjust her investment input choices because of the price change. Married mothers, by comparison, adjust their input choices more—by 0.50%. A subsidy to non-parental child care will cause them to shift more heavily towards that input than for single mothers. Fathers parenting in a couple have an intermediate elasticity: they are more responsive to the ratio of input prices than single mothers, but less responsive than their wives. This matters for policy analysis in general equilibrium, such as the one I conduct, because it is informative about what one can expect parents in the economy to do with their time in the presence of a subsidy. All parents will shift inputs away from their own time and toward non-parental child care if the latter's price decreases. Ceteris paribus, this shift will not be dramatic; to achieve large changes in time use, the subsidy will have to be large. Time use is relevant because if parents easily substituted away from parenting and (at least partially) into time spent working, this would be a source of expansion in the tax base. My estimates indicate that expansions in the tax base from parents substituting away from parenting quality time and into labor should be expected to be small. Changes in the tax base will have to come instead from changes in the population distribution of skill rather than changes in parental time use.

The third and final takeaway is the role of investment in the outermost aggregator, which takes investment and the current stock of skill to produce tomorrow's skill. The share parameter on investment is high: its role in producing tomorrow's skill dominates the role of the current stock of skill. In addition, the complementarity parameter between investment and skill is $\xi = -1.2$, which means that they are more complementary than allowed in a Cobb-Douglas parameterization. The corresponding elasticity of substitution is 0.45. Consider the multi-period estimation formulation presented above, and the intertemporal cost minimization equation that I use to derive my estimation equation for ξ. If the ratio of investment prices today and tomorrow changes by 1%, this estimate says that the ratio of investments today and tomorrow will change by 0.45%. The distribution of investments over time is not very sensitive to changes in the price of that investment across periods. In my model, unlike in the estimation data, there is a single period of childhood. In that context, there is no intertemporal cost minimization decision for parents. The interpretation of the complementarity parameter in a one-period technology is that it determines how quickly investment increases with the required adjustment to the current stock of skill. For example, if one wants to increase the current stock of skill by 10 percentage points, a skill accumulation technology that exhibits high complementarity (lower ξ) will require more investment than one with greater substitutability, for a given initial skill θ_c and share parameter υ.

3.5 Comparison with Other Findings

There is a large body of work on the estimation of skill technologies (e.g., see Todd and Wolpin (2003) and Cunha and Heckman (2008) for a discussion of specification and estimation issues). Direct comparisons are problematic because specifications vary across studies, but are nevertheless qualitatively informative. The data requirements vary with the estimation equations; mine require information on time inputs and their prices. One widely cited estimation is due to Cunha, Heckman, and Schennach (2010). In that study, the authors use the NLSY79 Children and Young Adults suvey to estimate a CES skill accumulation technology for a two-dimensional skill vector containing both cognitive and non-cognitive skills. Each of these dimensions of skill is allowed to affect the evolution of the other dimension. This skill technology is very general; it nests several specifications examined in other studies. Cunha, Heckman, and Schennach (2010) find substitutability between investment and skill during early childhood. In that study, the authors are not specific about which inputs aggregate into investment in skill.

Lee and Seshadri (2018) estimate a skill technology with a single dimension of skill, but assume that there is no initial draw of skill (or rather, that it is identical for everyone and equal to 0). They thereby impose by assumption a property that Cunha, Heckman, and Schennach (2010) found as a result: that in the first period of life, initial investment and the initial stock of skill are substitutable. For other periods, however, Lee and Seshadri (2018) are unable to reject a Cobb-Douglas specification. This is a higher level of substitutability than what I find, but lower than that in Cunha, Heckman, and Schennach (2010).[2]

My estimates show complementarity between investments and skill early in life. I do not ignore heterogeneity in the first measure I have of skill at 9 months, as in the specification of Lee and Seshadri (2018), because it doesn't appear to be noise in my data and because I have no observations of the investment that generated that initial stock (motivating regressions for these two claims are in the appendix). If the investment were solely responsible for the skill I observe at 9 months, the skill would already be correlated with family income. That is not what I find. See the appendix of Cunha, Heckman, and Schennach (2010) for a discussion of the implications of substitutability versus complementarity between skill and

[2]In their 2016 working paper, Agostinelli and Wiswall (2016) the authors show that the assumptions made in Cunha, Heckman, and Schennach (2010) (specifically re-normalization of the latent skill variables) impose over-identifying restrictions which can bias the estimation of the complementarity parameter. The direction of the bias is explored in Monte Carlo simulations which demonstrate that the direction of the bias depends on several attributes of the estimation procedure. My estimation method does not re-normalize skill distributions in each period. This is because by using intertemporal cost minimization to derive estimation equation (3.21), I am able to avoid using measures of skill in the estimation of the skill accumulation technology.

investment for policy design.

In my framework, children being raised in families with low income are more likely to be children of single mothers, who use a different technology than couples to invest in their children. This technology emphasizes non-parental child care more than the one couples use: the share on parental time is much lower. However, the ability of parents to adjust their own time use in response to a change in child care prices is higher for couples than for single mothers. On net, the larger share parameter dominates, so that subsidies targeted to single mothers target the families whose costs of investment are most sensitive to the price of non-parental child care. This heterogeneity across family structures is uncovered with my specification choice, unlike those used in other studies.

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Appendix A

A.1 Motivating Regressions for Model Specification

To motivate my model's specification, this section provides several regressions using ECLS-B data.

Table A.1: Predicting Initial Skill with Family Attributes + Prenatal Care

	(1) Married Couples	(2) Single Mothers	(3) All Mothers
Child is Female	0.174*	-0.0504	0.116*
	(0.0690)	(0.0871)	(0.0559)
B.A.: Mother	-0.0663	-0.271*	-0.130*
	(0.0752)	(0.124)	(0.0611)
B.A.: Father	-0.106		
	(0.0778)		
Hourly Wage: Mother	0.0393	-0.0792	0.0206
	(0.0338)	(0.0407)	(0.0328)
Hourly Wage: Father	0.0517		
	(0.0283)		
Month Began Prenatal Visits	0.0121	-0.00857	0.00789
	(0.0387)	(0.0425)	(0.0301)
No. Prenatal Visits	0.0166	-0.0192	-0.00254
	(0.0437)	(0.0490)	(0.0340)
Single Mother			0.0201
			(0.0614)
Constant	4.277***	4.693***	4.428***
	(0.163)	(0.194)	(0.131)
R^2	0.0151	0.0141	0.0085
N_sub	2900	1350	4250

Barely explains the variance. Units: standard deviations, except for indicators

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes rounded to nearest 50, following NCES requirements.

In Table A.1, I regress a child's test scores at age 4 on family attributes, child initial skill, and child gender. I do this separately for children of married couples and single mothers, because family attributes include those of the father for the former but not the latter (different explanatory variables).

Table A.2: Time Investments by Child Gender

	Married Couples		Single Mothers	
	(1)	(2)	(3)	(4)
	Tot. Parental Time	N Time	N Time	Total Time
Child is Female	-0.0845	-0.216	-1.156	0.0434
	(0.212)	(0.782)	(0.850)	(0.499)
Child Test Score [0,1]	8.528***	8.281***	1.023	9.945***
	(0.598)	(2.219)	(2.477)	(1.389)
B.A.: Father	2.104***	-0.895		
	(0.250)	(0.904)		
B.A.: Mother	1.850***	2.745**	1.578	3.484***
	(0.247)	(0.881)	(1.473)	(0.556)
Hourly Wage: Father	0.0105	-0.0248*		
	(0.00588)	(0.0118)		
Hourly Wage: Mother	0.00922*	-0.0146	-0.0638**	-0.0122
	(0.00369)	(0.0131)	(0.0244)	(0.0107)
Constant	-2.921***	10.59***	29.68***	18.85***
	(0.824)	(3.134)	(3.443)	(1.905)
R^2	.13	1.1e-02	5.0e-03	1.7e-02

Standard errors in parentheses. N stands for non-parental child care.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes rounded to nearest 50, following NCES requirements.

In Tables A.2 and A.3, I report regression analyses I use to motivate two modelling assumptions: parents do not target investments by child gender, and initial skill endowments (at 9 months) affect skill outcomes later in life (at 4 years of age). Table A.2 reports four models, each with a time input choice as the dependent variable. The first two are for married couples, the second two for single mothers. Time investments are predicted by attributes of the parents (hourly wages and educational attainment) and attributes of the child (current skill). Child gender is not a statistically significant predictor of parental time inputs, according to Table A.2 . There is some evidence in other studies that parenting behavior and treatment effects of the program vary by the gender of the child (see **?**, Kottelenberg and Lehrer (2014)), but I do not see this my analysis.

Table A.3: Predicting Final Skill with Initial Skill + Time Investments

	(1) Married Couples	(2) Single Mothers	(3) All
Initial Test Score (9 Mo.): Stdzd	0.138***	0.139***	0.130***
	(0.0389)	(0.0350)	(0.0373)
Hourly Wage: Mother	0.0565	0.217***	0.123**
	(0.0359)	(0.0500)	(0.0383)
Hourly Wage: Father	0.0971**		
	(0.0312)		
Child is Female	0.108	0.135	0.0941
	(0.0619)	(0.0738)	(0.0636)
B.A.: Mother	0.291***	0.662***	0.546***
	(0.0744)	(0.113)	(0.0651)
B.A.: Father	0.441***		
	(0.0788)		
Constant	0.851***	0.654***	1.005***
	(0.183)	(0.160)	(0.180)
R^2	0.1695	0.1273	0.1237
Observations	2900	1400	2900

Initial skill has predictive power. Units: standard deviations, except for indicators

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes rounded to nearest 50, following NCES requirements.

Table A.3 reports two models, one for married couples and one for single mothers. The dependent variable of both in both models is the final skill of the child at age 4. Explanatory variables include the initial skill of the child, gender of the child, indicators for parental educational attainment (BA or higher), and parental hourly wages. Initial test scores are statistically significant predictors for final test scores, and so are parental attributes related to their skill. This motivates including heterogeneity in initial skill endowments in my model.

Correlation of Child Skill and Family Income

Table A.4: Correlations of Skill and Family Income

	(1) Test Score W1	(2) Test Score W3
Family Income W1	0.000114	0.000651*
	(0.000103)	(0.000254)
Flag: Present in model 2 samplel	-0.00677	0
	(0.00754)	(.)
Family Income W3		0.000693***
		(0.000204)
Test Score W1 (SD)		0.141**
		(0.0482)
Constant	1.453***	1.004***
	(0.00733)	(0.0705)
R^2	.003	.125
Observations	1300	1500
Correlation	0.04	0.33
Correlation p-value	.35	0

Income in thousands of dollars. Test scores in standard deviation units.

Standard errors in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes rounded to nearest 50, following NCES requirements.

Table A.4 reports a slightly different version of the same qualitative points made with Tables A.1, A.2, and A.3. The dependent variables in the two models are initial skill and final skill, with both family structures pooled. Initial income at 9 months has no predictive power for the initial skill score. At age 4, however, final skill can be predicted with income (both at age 9 months and 4 years) and the initial test score. In this table I also report the correlations of the dependent variable for the model (initial skill for model 1, final skill for model 2) with family income in the same period (initial and final, respectively). These correlations jump from zero to 0.33. The latter number is my target for the correlation of child's skill at the end of childhood with family income. Note that the measure of family income I use here is income before labor income taxes but including transfers. I use the analogous object in the model in the internal calibration step.

A.2 Regression Results with Alternative Nestings

As discussed in the body of this paper, alternative nestings of the CES function imply different restrictions on the elasticity of substitution between inputs. In this section of the Appendix, I report skill accumulation technology estimation results analogous to the one reported in the text, but for different nestings of the technology (Tables A.5 and A.6). For each nesting, I first give the functional form it assumes. In the titles of the different specifications, M stands for mother, F stands for father, and N stands for non-parental care. I give the innermost nesting of the couple problem (F,M) or (F,N), followed by the outermost one (N, not N), (M, not M). The main qualitative points survive in the alternative nestings. The first alternative nesting presented below, however, does illustrate that restricting the substitutability of non-parental child care with mother and father time to be the same implies unrealistically large complementarity values for mother and father time.

Table A.5: Nesting: (F,M) + (N, not N)

	(1) Single Mother + Childcare	(2) Cohabiting Mother + Cohabiting Father	(3) Childcare + Not Childcare	(4) Outermost Estimation
$log(\frac{\text{after-tax hourly wage mother}}{\text{cost per hour child care}})$	-0.308***		-0.559***	
	(0.0856)		(0.0507)	
$log(\frac{\text{after-tax hourly wage father}}{\text{cost per hour child care}})$		-0.0547		
		(0.0349)		
log(composite term)			-0.514***	
			(0.120)	
$log(\frac{\Lambda_{type,t}}{\Lambda_{type,t-1}})$				0.469***
				(0.0292)
$log(\frac{r_{t-1}}{r_t})$				11.46***
				(0.653)
Constant	-1.117***	-0.427***	-0.179	0.729***
	(0.131)	(0.00543)	(0.107)	(0.0405)
R^2	.1356	.0022	.2294	.4566
Observations	1450	3100	3100	2600
complementarity	-2.25	-17.28	-.7895	-.8825
complementarity (se)	.9046	11.66	.1625	.1035
complementarity (p-value)	.0129	.1384	1.2e-06	1.5e-17
share	.0258	4.0e-04	.4207	.2534
share (se)	.0361	.0021	.052	.028
share (p-value)	.4749	.844	6.3e-16	1.6e-19

Standard errors in parentheses, clustered at the level of state of residence.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes are rounded to nearest 50, following NCES requirements.

Table A.6: Nesting: (F,N) + (M, not M)

	(1) Single Mother + Childcare	(2) Cohabiting Father + Childcare	(3) Cohabiting Mother + Not Cohabiting Mother	(4) Outermost Estimation
$log(\frac{\text{after-tax hourly wage mother}}{\text{cost per hour child care}})$	-0.298**	-0.363***		
	(0.0882)	(0.0657)		
log(composite term)		-0.815**		
		(0.232)		
$log(\frac{\text{after-tax hourly wage father}}{\text{cost per hour child care}})$			-0.445***	
			(0.0458)	
$log(\frac{\Lambda_{type,t}}{\Lambda_{type,t-1}})$				0.493***
				(0.0251)
$log(\frac{r_{t-1}}{r_t})$				11.80***
				(0.662)
Constant	-1.132***	-0.313**	-1.015***	0.746***
	(0.135)	(0.0940)	(0.0650)	(0.0430)
R^2	.1262	.227	.1406	.4925
Observations	1450	3100	3100	
complementarity	-2.351	-1.757	-1.248	-.9714
complementarity (se)	.9908	.4994	.2314	.0976
complementarity (p-value)	.0176	4.3e-04	7.0e-08	2.4e-23
share	.022	.2768	.0927	.2296
share (se)	.0339	.0607	.032	.0236
share (p-value)	.5157	5.2e-06	.0038	2.6e-22

Standard errors in parentheses, clustered at the level of state of residence.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sample sizes are rounded to nearest 50, following NCES requirements.

A.3 Description of Data Sources

The Early Childhood Education Longitudinal Study, Birth Cohort

The ECLS-B follows a nationally representative sample of families with a child who was 9 months old in 2001. It was designed and collected by the United States Department of Education. Using birth-certificate data from the National Center for Health Statistics, over 14,000 births were selected within Primary Sampling Units. Children of mothers younger than 15 were excluded from the sampling frame. There are 5 waves: wave 1 is the 9-month old data collection round, wave 2 occurs at 2 years, wave 3 at 4 years, and waves 4 and 5 at kindergarten entry. If the focal child was not in kindergarten when wave 4 was collected, the surveyors went back and collected data the next year when they were enrolled. In addition, if a child repeated kindergarten, their scores were also collected in wave 5 in addition to wave 4. Each wave contains several instruments; these are different self-administered questionnaires (SAQs) for different people in the child's life, in addition to the child-level data. Table 22 summarizes these instruments in each wave of the survey.

Table A.7: The Structure of the ECLS-B

Instrument	Wave 1	Wave 2	Wave 3	Wave 4+5
1.	Parent Interview	Parent Interview + SAQ	Parent SAQ	Parent SAQ
2.	Resident Father	Resident Father SAQ	Resident Father SAQ	ECEP Interview[1]
3.	Nonresident Father	Nonresident Father SAQ	Preschool Center Director SAQ	Teacher
4.		Child Care Provider	Preschool ECEP SAQ[1]	WECEP Interview [2]
5.		Center Director		

[1] Early Care and Education Provider
[2] Wrap-around Care Early Care and Education Provider

In each wave of the survey, the primary care provider (usually the mother) and the resident father fill out detailed questionnaires on the activities they do with their kids and at what frequency (once a week, twice a week, once a month, etc.). In addition, they report age, educational attainment, income, hours worked, the number of hours the child spent in non-parental care, what type of care that was (relative, non-relative, center-based), and the cost of that care.

I define quality time as the total amount of time spent (1) reading to the child (2) talking with or listening to the child (3) singing to the child. To map from frequencies of activities to levels of quality time supplied by parents, I impute amount of time per activity using data from the ATUS. The imputation uses common characteristics observed across both samples: gender, marital status (married/cohabiting or single), labor force status, and educational attainment. Here educational attainment is less than a college degree, or a college degree or more. For hourly wages, I use time spent working and income to compute the pre-tax levels, and then Table 2 of McGrattan and Prescott (2017) to correct for labor income taxes. For hourly prices of non-parental care, I use total cost of child care and total hours in child care for the primary source of non-parental care reported by the primary caregiver of the survey child subject.

American Time Use Survey

I use a pooled sample from the ATUS from 2003 to 2016, which is provided with CPS variables on age, gender, marital status, labor force status, educational attainment and family structure along with a time diary. I restrict the sample to parents who are between 15 and 55, with a child 4 years or younger. I use information on gender, marital status (married/cohabiting or single), labor force status, and educational attainment, where educational attainment is recoded as less than a college degree or a college degree and higher.

Imputation

In the baseline estimation, I focus on parents of children under 4 years old who are aged 55 and younger. In the ECLS-B this is respondents of waves 1 to 3. I additionally restrict observations to those who report age, educational attainment, and how often they participated in the activities of interest explained above. I further restrict the sample by excluding outliers: I only use those observations for whom after-tax hourly wages are greater than 0.1 cents and no greater than 200 dollars an hour. Finally, I only consider families observed in each and all of the first three waves of the survey, so that a family-specific mean can be computed for the fixed effects estimator.
Given these characteristics, I find survey-weighted averages of time spent on an activity for those who report engaging in it. The activities whose average duration I tabulate are (1) time spend reading to your child (2) time spent talking and listening. I assign the level of time spent reading and talking and listening to their respective activities in the ECLS-B; additionally, I assign the level of time spent talking and listening to the singing activity reported by parents in the ECLS-B. I do this because there is no singing activity reported in the ATUS.
Once I have imputed time levels per activity, I aggregate levels of quality time per parent in each family in each wave. This gives me quality time investments, non-parental time investments, after-tax hourly wages, and non-parental child care prices at the child-family pair and wave level. With this information, I can implement my estimation.

Summary Statistics for Raw and Estimation Sample in ECLS-B and Imputation Sample in ATUS

In the following two subsections I report sample summary statistics for the raw data and the estimation sample. For the raw data I report by wave of the sample, and for the estimation sample I pool all the viable observations in the three waves and report statistics on those. I also report summary statistics for the ATUS sample I used to impute time levels to observed frequencies of activities in the ECLS-B sample.

Summary Statistics ECLS-B

The following summary statistics describe the ECLS-B data, pooled across family structures after the imputation from the ATUS, by wave and before data cleaning. The population moments I use in the internal calibration for the fraction of parents who are single mothers comes from this sample. The fraction in the sample that also reports variables necessary for estimation is larger than these population moments. Notice that the fraction below 185% of the poverty line in the pooled sample is quite high, at 50%. This drop to 40% by the time the child is age 4. Averages here include observations for which the response is 0. This explains why the average age of the father is now lower than the mother's.

Table A.8: Summary Statistics Raw Sample ECLS-B: Wave 1

	Wave 1				
	Obs.	mean	sd	min	max
Education Time Father: Hours per Week	10700	2.88	3.37	0.00	12.67
Education Time Mother: Hours per Week	10700	7.43	3.15	0.00	12.75
Below Poverty Line	10700	0.26	0.44	0.00	1.00
Below 185% of Poverty Line	10700	0.50	0.50	0.00	1.00
Marriage Rate	10700	0.64	0.48	0.00	1.00
Cohabitation Rate	10700	0.78	0.42	0.00	1.00
Single Mother	10700	0.21	0.40	0.00	1.00
Nonparental Care: Hours per Week	10700	14.32	18.53	0.00	70.00
Ratio of Time: Father/Mother	10400	0.44	0.64	0.00	9.93
Ratio of Time: Mother/Childcare	5200	0.54	0.96	0.00	12.75
Hourly Pay Mother, After Tax	5000	11.13	21.46	0.00	788.00
Hourly Pay Father, After Tax	6600	13.43	17.12	0.00	827.40
$\frac{\text{Cost per hour childcare}}{\text{After-tax hourly wage mother}}$	3500	3.41	177.25	0.00	10479.04
Hourly Cost Childcare	8250	1.25	3.37	0.00	99.50
Age Res. Mother	10700	28.46	6.64	-1.00	68.00
Age Res. Father	10700	25.16	14.83	-9.00	75.00
Mother: BA or higher	10700	0.26	0.44	0.00	1.00
Father: BA or higher	10700	0.25	0.43	0.00	1.00

Sample sizes rounded to nearest 50, following NCES requirements.

Table A.9: Summary Statistics Raw Sample ECLS-B: Wave 2

	Wave 2				
	Obs.	mean	sd	min	max
Education Time Father: Hours per Week	10700	3.41	4.12	0.00	12.67
Education Time Mother: Hours per Week	10700	7.94	4.04	0.00	12.75
Below Poverty Line	10700	0.00	0.00	0.00	0.00
Below 185% of Poverty Line	10700	0.00	0.00	0.00	0.00
Marriage Rate	10700	0.59	0.49	0.00	1.00
Cohabitation Rate	10700	0.69	0.46	0.00	1.00
Single Mother	10700	0.19	0.39	0.00	1.00
Nonparental Care: Hours per Week	10700	14.04	18.51	0.00	80.00
Ratio of Time: Father/Mother	9350	0.46	0.57	0.00	9.83
Ratio of Time: Mother/Childcare	4800	0.48	0.76	0.00	12.67
Hourly Pay Mother, After Tax	5100	11.64	26.28	0.00	1004.70
Hourly Pay Father, After Tax	6200	15.14	52.93	0.00	3546.00
$\frac{\text{Cost per hour childcare}}{\text{After-tax hourly wage mother}}$	3900	348.68	21649.58	0.00	1345064.75
Hourly Cost Childcare	9000	1.05	2.31	0.00	64.00
Age Res. Mother	9850	29.79	7.02	-1.00	70.00
Age Res. Father	9850	26.20	15.38	-9.00	76.00
Mother: BA or higher	10700	0.33	0.47	0.00	1.00
Father: BA or higher	10700	0.32	0.47	0.00	1.00

Sample sizes rounded to nearest 50, following NCES requirements.

Table A.10: Summary Statistics Raw Sample ECLS-B: Wave 3

	Wave 3				
	Obs.	mean	sd	min	max
Education Time Father: Hours per Week	10700	2.22	2.80	0.00	12.67
Education Time Mother: Hours per Week	10700	5.15	3.78	0.00	12.75
Below Poverty Line	10700	0.20	0.40	0.00	1.00
Below 185% of Poverty Line	10700	0.39	0.49	0.00	1.00
Marriage Rate	10700	0.52	0.50	0.00	1.00
Cohabitation Rate	10700	0.59	0.49	0.00	1.00
Single Mother	10700	0.17	0.37	0.00	1.00
Nonparental Care: Hours per Week	10700	16.53	16.52	0.00	96.00
Ratio of Time: Father/Mother	8100	0.48	0.54	0.00	6.57
Ratio of Time: Mother/Childcare	7100	0.44	0.60	0.00	12.67
Hourly Pay Mother, After Tax	5000	12.58	23.18	0.00	1063.80
Hourly Pay Father, After Tax	5400	15.95	16.97	0.00	506.57
$\frac{\text{Cost per hour childcare}}{\text{After-tax hourly wage mother}}$	3200	253.07	10139.75	0.00	419839.25
Hourly Cost Childcare	7500	2.19	3.83	0.00	64.71
Age Res. Mother	9000	32.28	7.56	-1.00	82.00
Age Res. Father	9000	28.16	16.36	-9.00	83.00
Mother: BA or higher	10700	0.41	0.49	0.00	1.00
Father: BA or higher	10700	0.39	0.49	0.00	1.00

Sample sizes rounded to nearest 50, following NCES requirements.

ATUS Imputation Data

For the imputation of time levels per activity, I used a pooled sample from the ATUS 2003-2016. I divide parents of children 3 and under into bins by labor force status, educational attainment, marital status and gender. For each bin, I find the average time spent reading, doing educational activities, and playing with each bin's children conditional on having done each activity during the observation period. This is the level I assign to the ECLS-B sample for time spent talking and listening or time spent singing (educational hours) or time spent reading (reading hours). Table A.11 displays group averages.

The population of these groups in the ATUS, even over such a large sample period, can be quite small. For example, only 85 married men not in the labor force with less than a BA reported reading to their children under 3 between 2003-2016. There were 35 single women with greater than a college degree (BA), not in the labor force, who reported spending educational time with their children aged 3 and under.

Table A.11: ATUS Imputation: Group Means (hours per week)

Group	Reading Hours	Reading Obs	Educational Hours	Educational Obs
Married Male in LF lt BA	0.512	612	0.459	371
Married Male in LF gte BA	0.468	1318	0.528	337
Married Male not in LF lt BA	0.694	85	0.359	43
Married Female in LF lt BA	0.457	778	0.603	472
Married Female in LF gte BA	0.458	1799	0.565	601
Married Female not in LF lt BA	0.556	860	0.640	580
Married Female not in LF gte BA	0.565	1062	0.521	413
Single Female in LF lt BA	0.458	273	0.541	198
Single Female in LF gte BA	0.361	134	0.326	44
Single Female not in LF lt BA	0.501	195	0.670	194
Single Female not in LF gte BA	0.621	34	0.409	35

Measures of Skill in the ECLS-B

There are three skill measures from the ECLS-B that I use in the regressions presented in Tables 16 to 18, one for waves 1,2 and 3. In wave 1, the ECLS-B reports test scores from the Bayley Short Form - Research Edition, which is a shortened version of the Bayley Scaled of Infant Development, Second Edition (BSF-R and BSID-II, respectively). The latter exam is the standard one for measuring development in children under 42 months of age. The BSF-R is a shortened version of the BSID-II, asking only some of the questions. Its scores are then re-scaled to make them comparable with scores from children who receive the BSID-II. For the initial test score variable used in Tables 16 to 19, I take the scale scores of the BSF-R at 9 months (in the first wave of the ECLS-B), which are reported both for mental and motor development. I then take the average of the two, and next I standardize them to lie between 0 and 1. For the test score value in the second wave, used in Table 17, I do the same procedure with the BSF-R scores recorded at 2 years of age (in wave 2 of the ECLS-B). By age 5, when the child is 48 months, the BSID-II and its subset exam the BSF-R are no longer an age-appropriate measures of development for children. Instead, the ECLS-B reports a new assessment battery that covers cognitive development in the domains of language, literacy, color knowledge, and mathematics. This is reported as the ECLS-B Direct Cognitive Assessment in several formats. I use the overall scale score of the Direct Cognitive Assessment and standardize it to lie between 0 and 1. This is the final test score used in the regressions of Table 18.

A.4 Child Support Payments from Single Fathers and Transfers as a Fraction of GDP

To discipline child support payments from single fathers, I use Table 3.12 from NIPA, Census tabulations on family counts in 2001, and the US Census Bureau report ”Custodial Mothers and Fathers and Their Child Support: 2001”. From US Census counts, I set the number of families in the US in 2001 to 71,787,347. Table 1.1.5 from NIPA gives the total amount of transfers in 2001 by components: I sum federal benefits from social insurance funds, Supplemental Nutrition Assistance Program (SNAP), supplemental security income, and refundable tax credits and other (which includes payments to nonprofit instutitions and student loans, among other categories). This totals 815 billion dollars. Combined with the number of families, this gives about 11,354 dollars per family in transfers in 2001. From the Census Bureau report, the average level of child support received by custodial mothers was 3,160 dollars in 2001. This makes the average child support payment due as a fraction of the average transfer to families about 0.28. Transfers as a fraction of GDP use Table 1.1.5 from NIPA in addition to Table 3.12. Total GDP in 2001 was 10,581 in billions of dollars. After rounding, this makes transfers as a fraction of 8% of GDP.

A.5 Labor Supply in the CPS

To calculate the average labor supply for the data moment in the internal calibration, I use Annual Social and Economic Supplement of the CPS from 2000. I define hours worked per week as the self-reported hours worked last week. I assign a 0 for this value for those who report not being in the labor force. I then find the average hours worked per week, weighted by the supplement household weight, for those between the ages of 20 and 80 with children under 5 whose marital status is reported as either “never married/single” or “married, spouse present”. This is a sample of 11,771 individuals, 53% of whom are women and 88% of whom are married. The resulting moment for average labor supply in this group is 31 hours per week, or 0.31 when expressed as a fraction of 100 disposable hours per week.

A.6 Earnings Distribution: Model v. Data

In Table A.12, I show how the implied earnings distribution of the baseline equilibrium compares with that found in the CPS for 2001 by **?**, referred to as “HSV” in the table. The p50/p10 ratio is quite close in the comparison, but the moments involving the 90th percentile are quite different in my model compared to the data. This is because the data is characterized by a tail in the income distribution, so that the 90th percentile of earnings is quite high relative to other percentiles in the distribution. In my model, by contrast, the 90th percentile is not that much higher than the median earnings level, and so the 90/50 ratio is small relative to it's empirical counterpart.

Table A.12: Untargeted Moments (Model Fit)

Moment	Source	Data	Model
$\frac{\text{p50}}{\text{p10}}$	HSV (CPS)	3.2	2.9
$\frac{\text{p90}}{\text{p50}}$	HSV (CPS)	2.4	1.5
Gini	HSV (CPS)	0.41	0.27

Notes: Table A.12 compares moments of the empirical earnings distribution with the model baseline equilibrium.

A.7 Spending on Child Care in the PSID

A common argument in the investment aggregator, instead of child care time, is goods (or money) spent on the child (examples include Lee and Seshadri (2018), Daruich (2017) and Abbott (2018). In this section, I use tabulations from the 2001 PSID and 2002 PSID CDS to show how child care expenses contribute to total expenditures on the child. To do this, I construct four different measures of total expenditures on the child (Definitions 1 to 4 in the tables below, with each definition specified in the table footnote). Next, I find the fraction of each measure of total expenditures that comes from child care. My conclusion is that child care represents the main component of the expenditures on children in the PSID. In that sense, using time in non-parental child care as an input, and including expenditures on child care in the budget constraint of parents, can be viewed as narrowing in on the main component of expenditures on children and being specific about how it contributes to child skill accumulation (through time use).

Table A.13: Definition 1

	mean	sd	count
Ages [0,3]	0.67	0.29	84
Ages [0,5]	0.68	0.28	146
Ages [0,7]	0.71	0.26	223
Ages [0,9]	0.70	0.27	260
Ages [0,11]	0.70	0.27	275

Notes: Table A.13 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 1 of total expenditures on children includes child care, money spent on toys, and money spent on school supplies

Table A.14: Definition 2

	mean	sd	count
Ages [0,3]	0.55	0.28	84
Ages [0,5]	0.58	0.28	146
Ages [0,7]	0.61	0.26	223
Ages [0,9]	0.60	0.27	260
Ages [0,11]	0.60	0.27	275

Notes: Table A.14 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 2 of total expenditures on children includes child care, money spent on toys, and money spent on school supplies.

Table A.15: Definition 3

	mean	sd	count
Ages [0,3]	0.49	0.27	83
Ages [0,5]	0.52	0.27	144
Ages [0,7]	0.55	0.26	220
Ages [0,9]	0.53	0.26	256
Ages [0,11]	0.53	0.26	271

Notes: Table A.15 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 3 of total expenditures on children includes child care, money spent on toys, money spent on school supplies, money spent on vacations, and money spent on clothes.

Table A.16: Definition 4

	mean	sd	count
Ages [0,3]	0.42	0.24	71
Ages [0,5]	0.43	0.24	126
Ages [0,7]	0.46	0.24	194
Ages [0,9]	0.44	0.23	229
Ages [0,11]	0.43	0.23	243

Notes: Table A.16 presents averages by age group for the fraction of total expenditure on children spent on child care. Definition 3 of total expenditures on children includes child care, money spent on toys, money spent on school supplies, money spent on vacations, money spent on clothes, and money spent on food.

A.8 Single Fathers in the ECLS-B

The ECLS-B provides a non-resident father questionnaire (NRQ) in the first two waves of the survey. In this section, I document six points about the sample of non-resident fathers that complete this survey as well as attributes of single mothers in the data. Sample counts for these tabulations reflect response rates for the questions of interest; here, I am not restricting by whether I also observe variables necessary for the skill accumulation technology estimation. In the statistics presented below, I use survey weights for the primary caregiver sample in wave 2. The main purpose of this section is to establish that relatiely few single fathers complete the survey, that those who do are not representative of the sample of single fathers, and that when they do complete the survey their answers and the answer's of their child's mother do not coincide. In addition, Table 33 makes an additional point about the marital status composition of single mothers: most were never married. This coincides with the timing and nature of the marriage market in my model.

The first three points are made in Tables A.17 and **??**. First, Table A.17 shows that the response rate of non-resident fathers in each wave is about 1 in 3. Second, Table **??** shows that the marital status of the corresponding single mother is about the same for the group of families with a completed NRQ and without a completed NRQ. Third, Table **??** also shows that most single mothers were never married (about 70% and 65% in the first and second wave of the survey, respectively). Since I do not model divorce, the composition of marital status in single mothers is important to check.

Table A.17: Response Rate NR Questionnaire

	(1) Wave 1	(2) Wave 2
Yes	0.300	0.309
No: Refusal	0.292	0.179
No: Not Permission	0.194	0.270
No: Ineligible, Lack of Contact	0.184	0.179
No: no NR	0.0290	0.0596
No: P not Biomother	0.000628	0.00303
Total	1	1
Obs.	2000	2000

Table A.17 displays response rates of non-resident fathers to the non-resident father survey in the ECLS-B. Slightly less than one-third of non-resident fathers respond. Sample sizes rounded to nearest 50, following NCES requirements.

Table A.18: Marital Status Composition of Mothers with NR fathers, by Questionnaire Response status

	Wave 1		Wave 2	
	(1) Completed NRQ	(2) No NRQ	(3) Completed NRQ	(4) No NRQ
Not Reported	0	0.00369	0	0.000118
Married	0.0640	0.0520	0.0921	0.0980
Separated	0.107	0.119	0.105	0.0939
Divorced	0.0909	0.0929	0.111	0.140
Widowed	0.00320	0.0142	0.00195	0.0169
Never Married	0.734	0.717	0.691	0.647
Not Biomother or Adoptive Parent	0	0.000897	0	0.00439
Total	1	1	1	1
Obs.	650	1350	650	1400

Table **??** displays the marital status composition of families where the biological parents are not cohabiting (single-parent families). The compositions are broken down by response status for the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

The next three points are made in Tables A.19-A.23. For point four, Table A.19 tabulates the days since the non-resident father last saw the child. Fathers who complete the NRQ have seen the child on average 1.5 days more recently than fathers who do not. Fifth, in Table A.19 I tabulate responses to the question "In a typical week, does [the child's] father spend a lot, some, very little, or no time taking care of [the child]?", for families without a completed NRQ's (first column) and for those with an NRQ (second column). Fathers who completed the NRQ are more likely to be parenting with a resident primary caregiver who responds "A lot" to this question (35% compared to 12%). Relatedly, Table A.21 shows that fathers who complete the NRQ are far more likely to have seen their child in the last month than fathers who did not complete the NRQ (90% versus 46%). Sixth, in Tables A.22 and A.23 I tabulate the wave 2 responses to the question "When it comes to making major decisions, please tell me if [child's] father has no influence, some influence, or a great deal of incluence on such matters as child care?", separately for mothers (Table A.22) in families without an NRQ (column 1) and those with an NRQ (column 2) and fathers (Table

A.23) who completed the NRQ. Fathers who completed the NRQ think they have a lot of influence; mothers with children whose fathers completed the NRQ say they have less influence than the fathers claim, although they report more influence more than do mothers in families without a completed NRQ.

Table A.19: Wave 1: Number of Days since NRF last saw child

	(1) No NRQ	(2) Completed NRQ
No. Days	3.860	2.353
Obs.	1300	650

Table A.19 displays the average number of days since a non-resident father saw his child in the first wave of the survey, by response status to the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table A.20: Wave 2: Frequency NRF last provides child care

	(1) No Completed NRQ	(2) Completed NRQ
Not Applicable	0.541	0.104
A lot	0.121	0.350
Some	0.117	0.280
Very little	0.0912	0.146
No time	0.130	0.120
Total	1	1
Obs.	1350	650

Table A.20 displays the response to the question: "In a typical week, does [the child's] father spend a lot, some, very little, or no time taking care of [the child]?", for families without a completed NRQ's (first column) and for those with an NRQ (second column). Sample sizes rounded to nearest 50, following NCES requirements.

Table A.21: Wave 2: Number of Days since NRF last saw child

	(1) No NRQ	(2) Completed NRQ
Don't Know	0.01	0
Refused	0.01	0
Not Applicable	0.06	0
Less than 1 month	0.459	0.896
More than 1 month, less than 1 yr	0.238	0.0718
More than 1 yr	0.0975	0.0198
No contact since birth/separation	0.133	0.0118
Total	1	1
Obs.	1350	650

Table A.21 compares the amount of time since non-resident fathers last saw their child, by response status to the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table A.22: Wave 2: Mother's Opinion of Father's Influence on CC

	(1) No NRQ	(2) Completed NRQ
Not Applicable	0.373	0.0207
No Influence	0.341	0.395
Some Influence	0.138	0.260
A Great Deal of Influence	0.148	0.324
Total	1	1
Obs.	1400	650

Table A.22 tabulates mother's responses to the question: "When it comes to making major decisions, please tell me if [child's] father has no influence, some influence, or a great deal of incluence on such matters as child care?", by response status for the non-resident father questionnaire. Sample sizes rounded to nearest 50, following NCES requirements.

Table A.23: Wave 2: NRQ Father's Opinion of Father's Influence on CC

	Frequency
Not Ascertained	0.0318
No Influence	0.146
Some Influence	0.328
A Great Deal of Influence	0.494
Total	1
Obs.	650

Table A.23 tabulates the response of father's who completed the non-resident father questionnaire to the question "When it comes to making major decisions, please tell me if you have has no influence, some influence, or a great deal of incluence on such matters as child care?". Sample sizes rounded to nearest 50, following NCES requirements.